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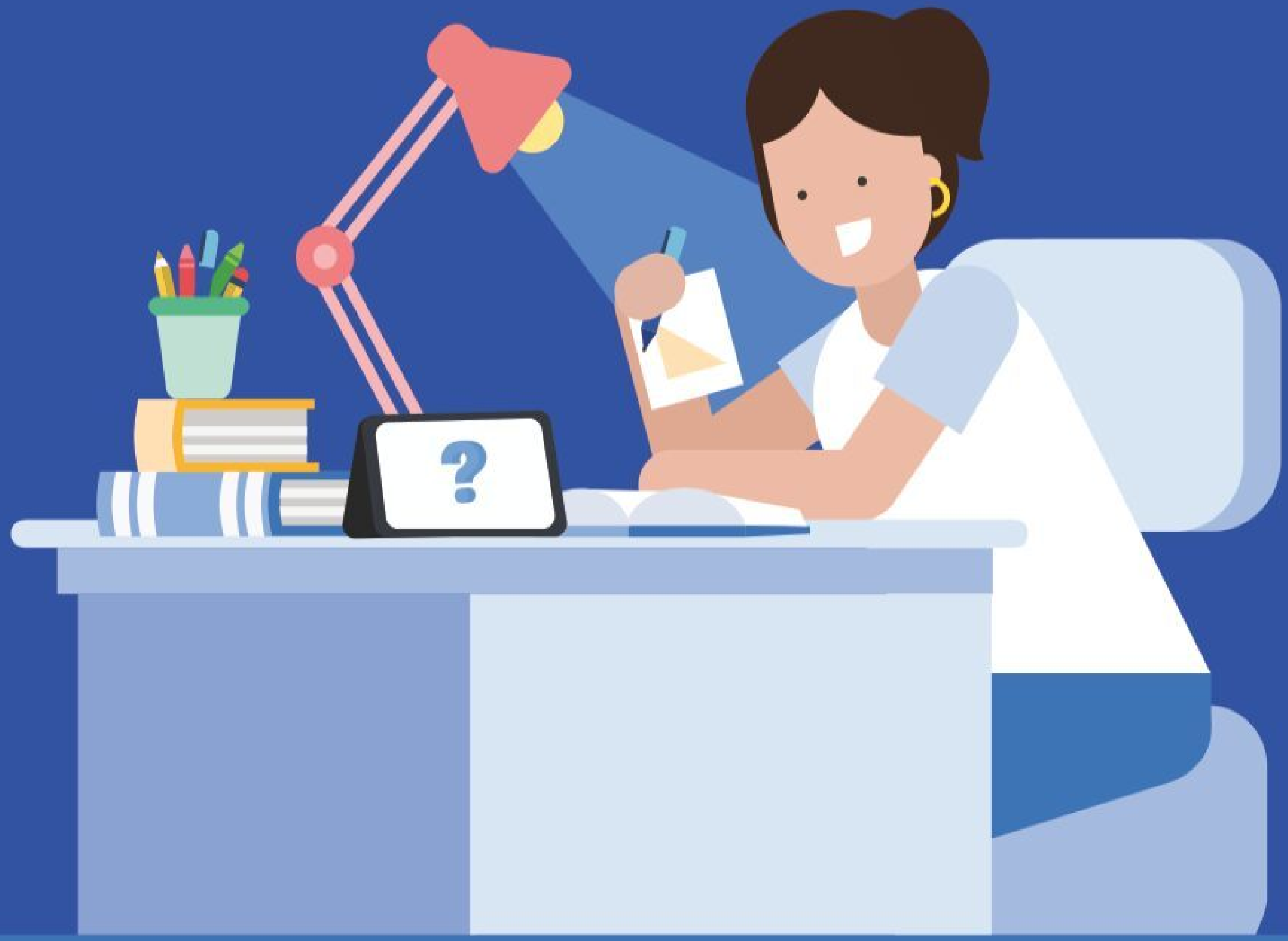

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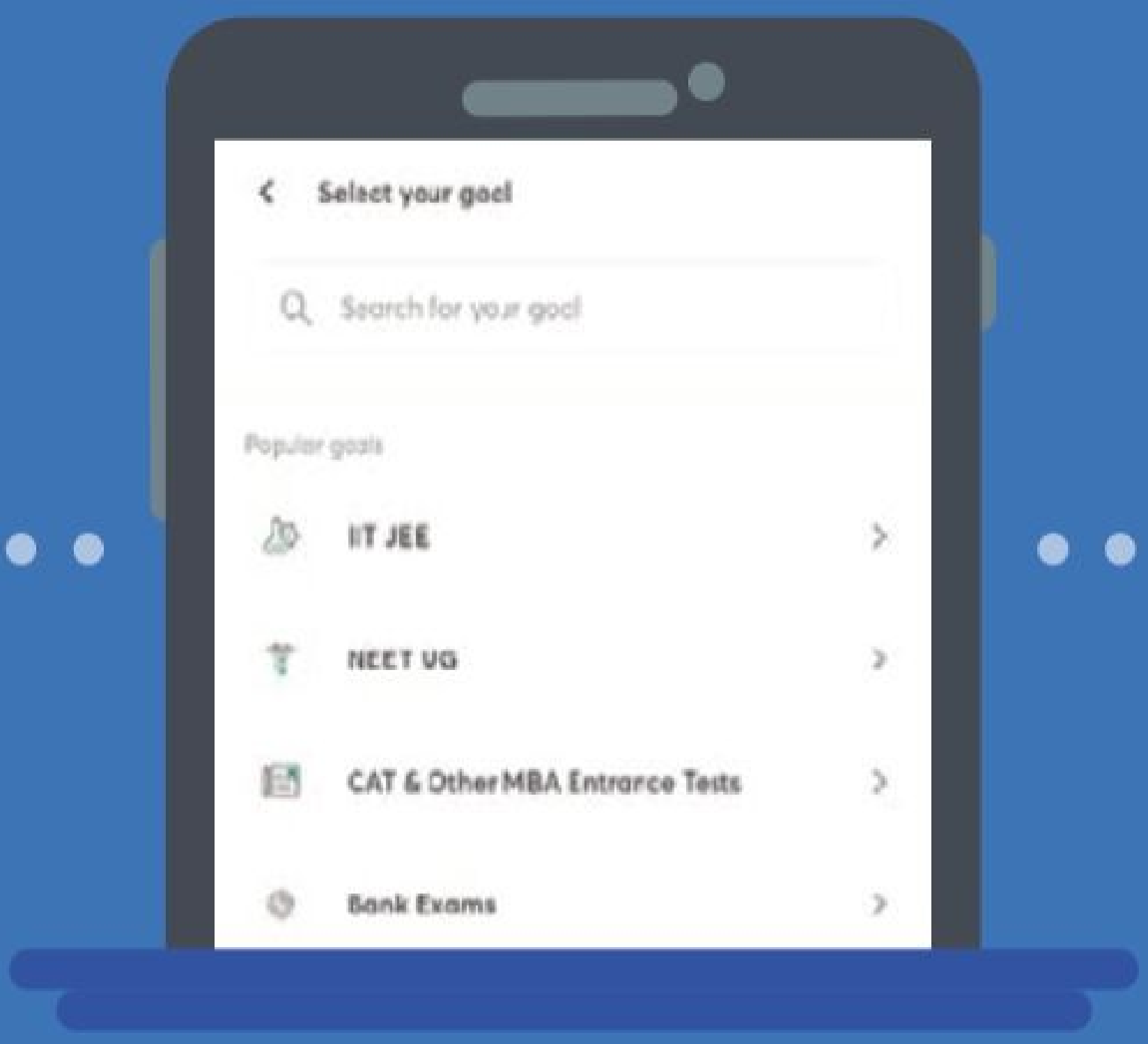

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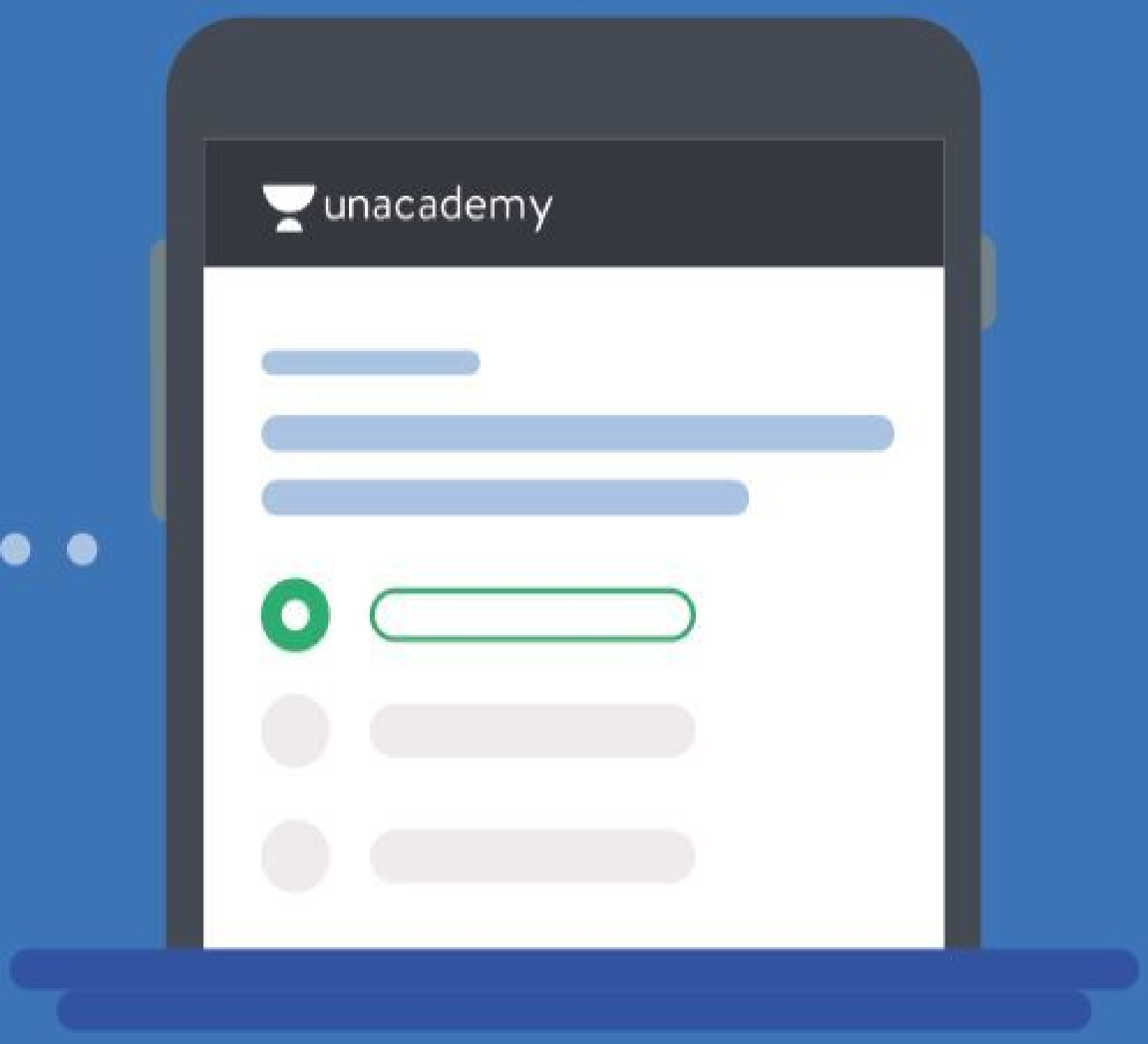

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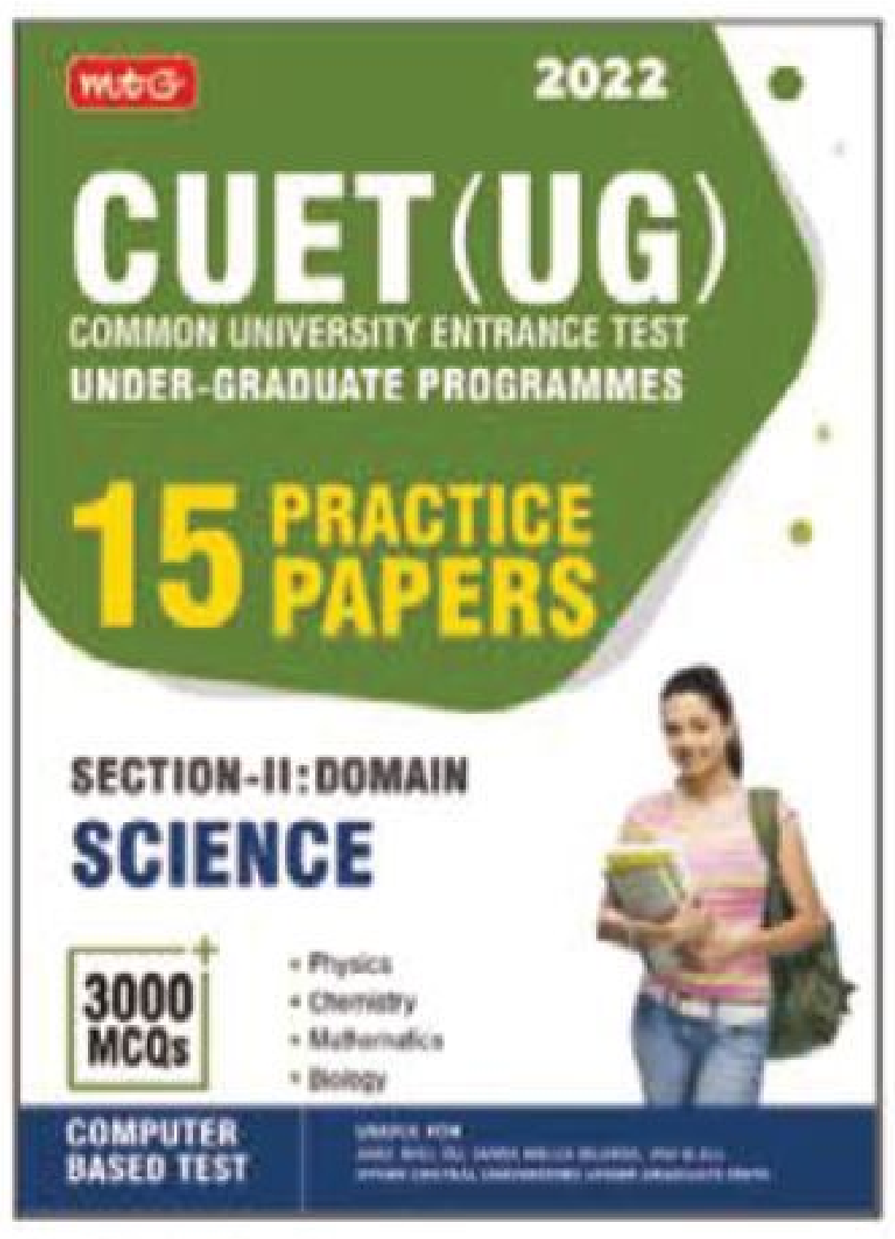
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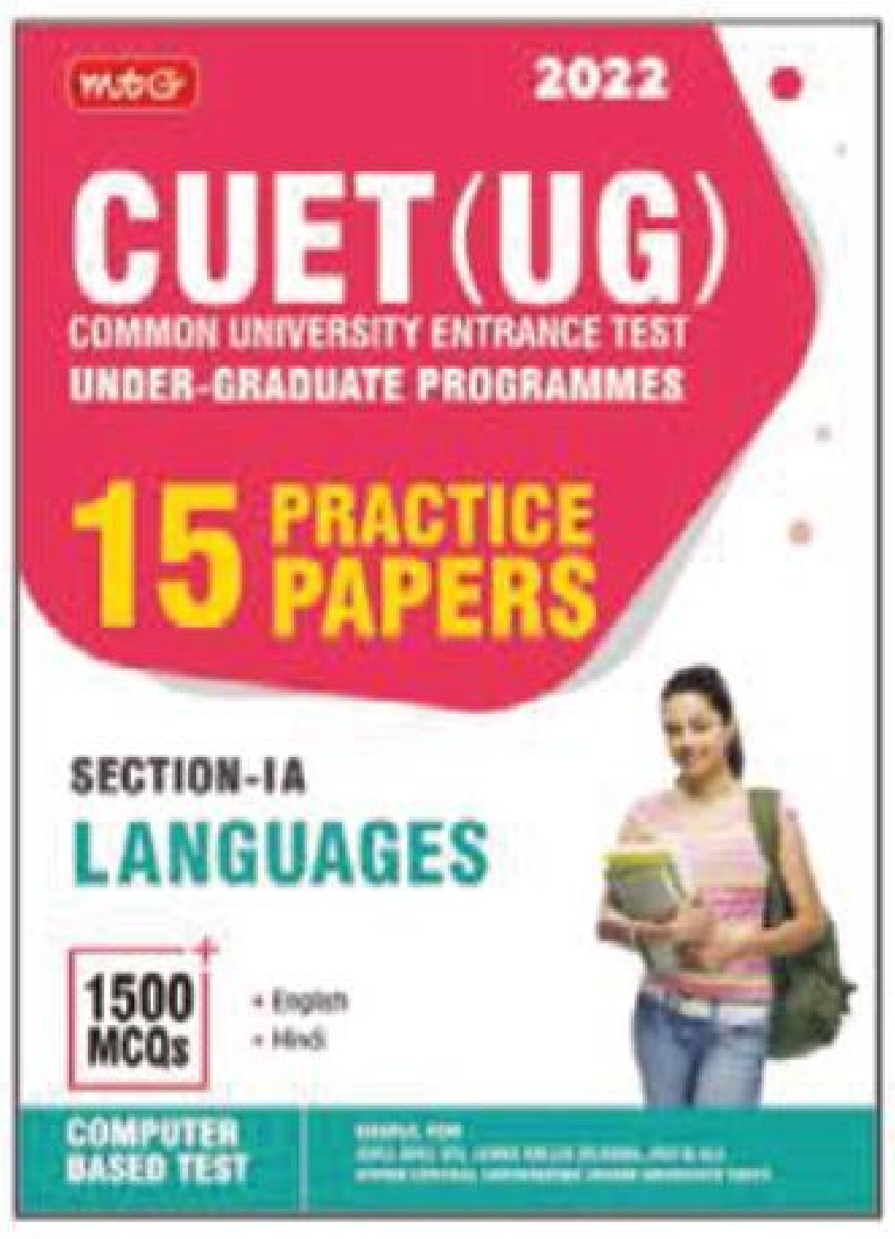
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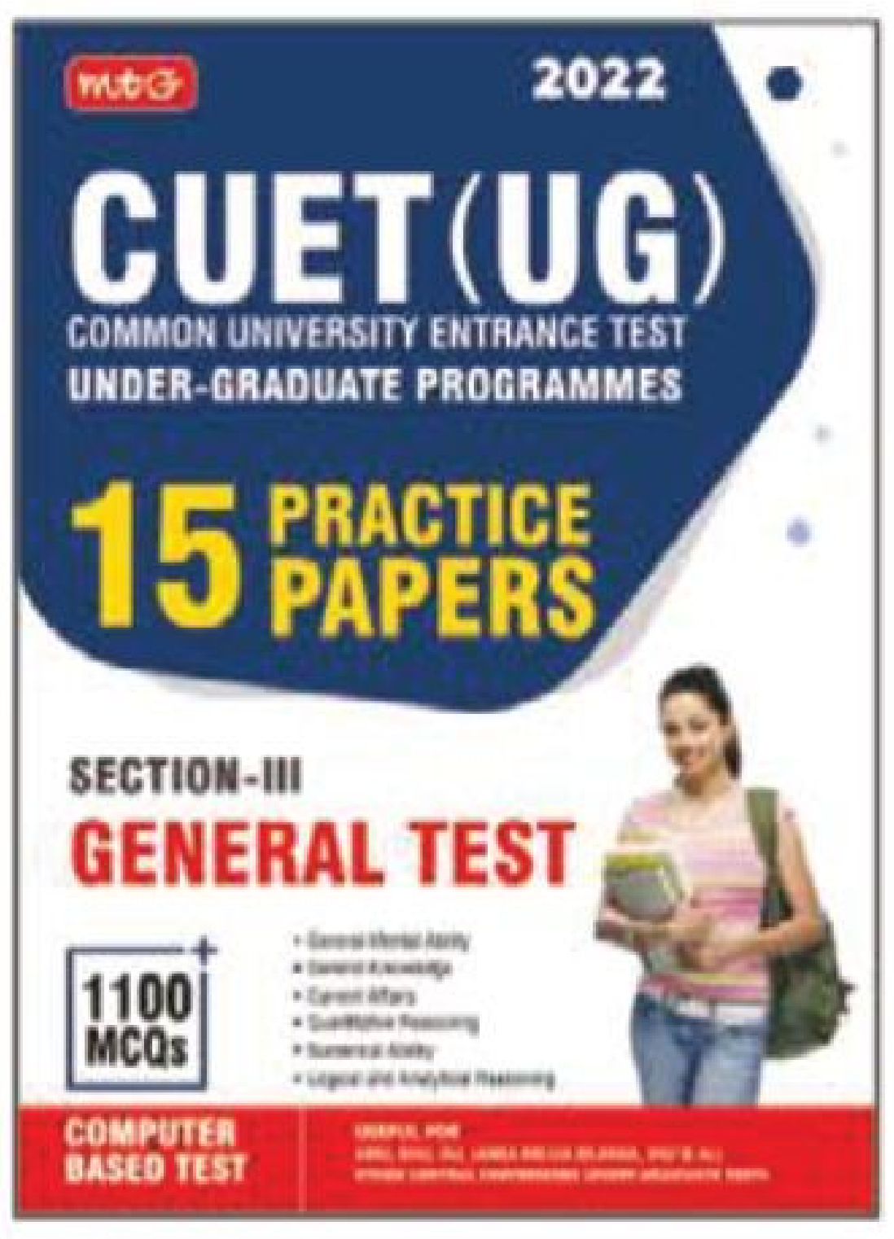
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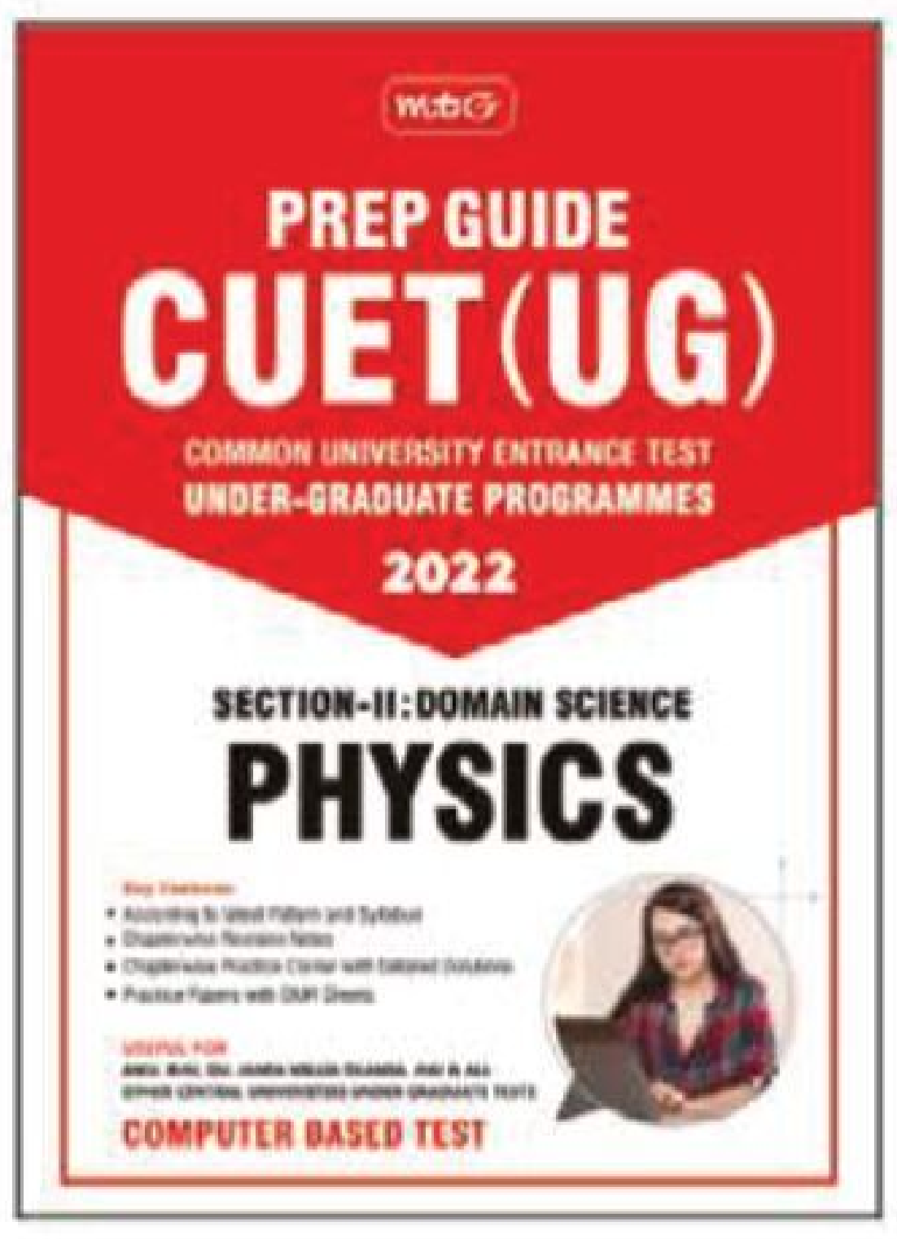
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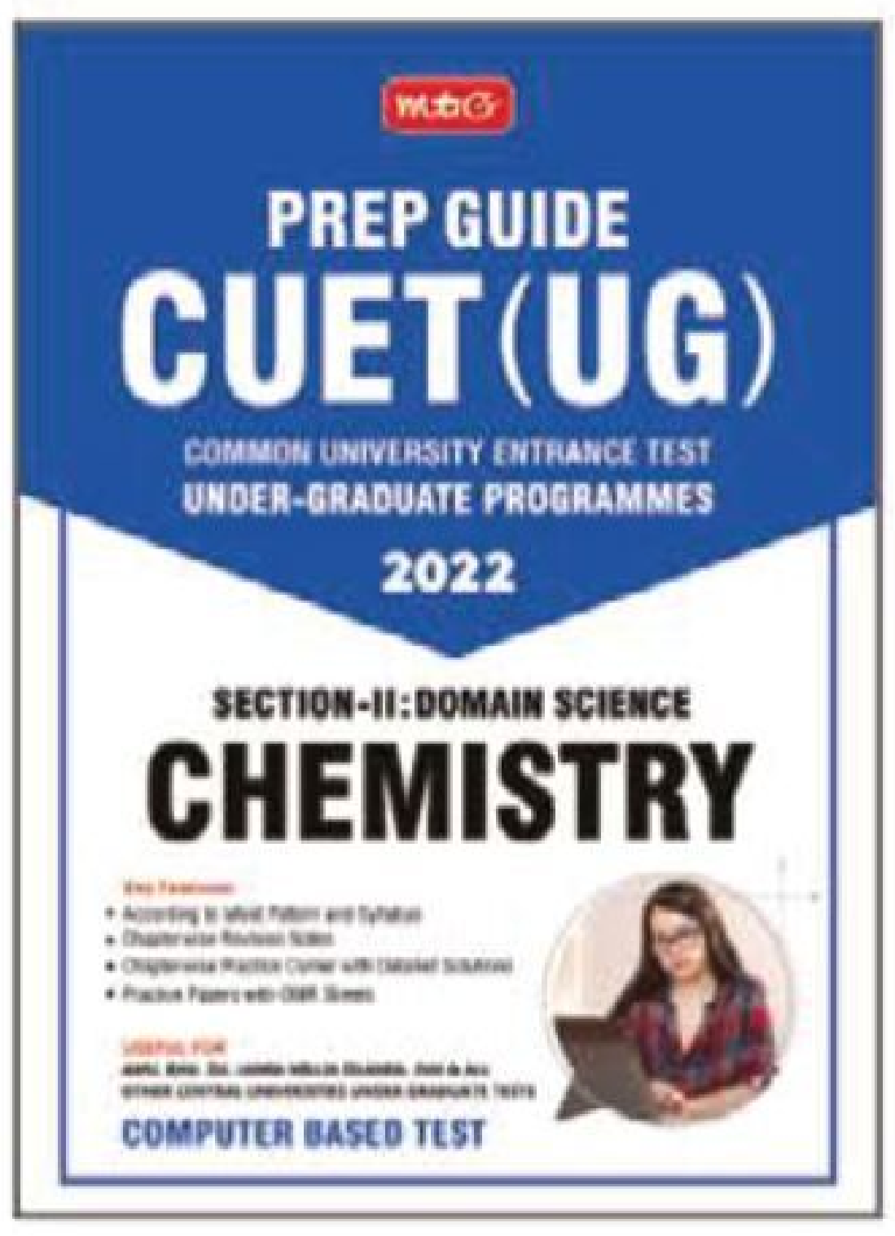
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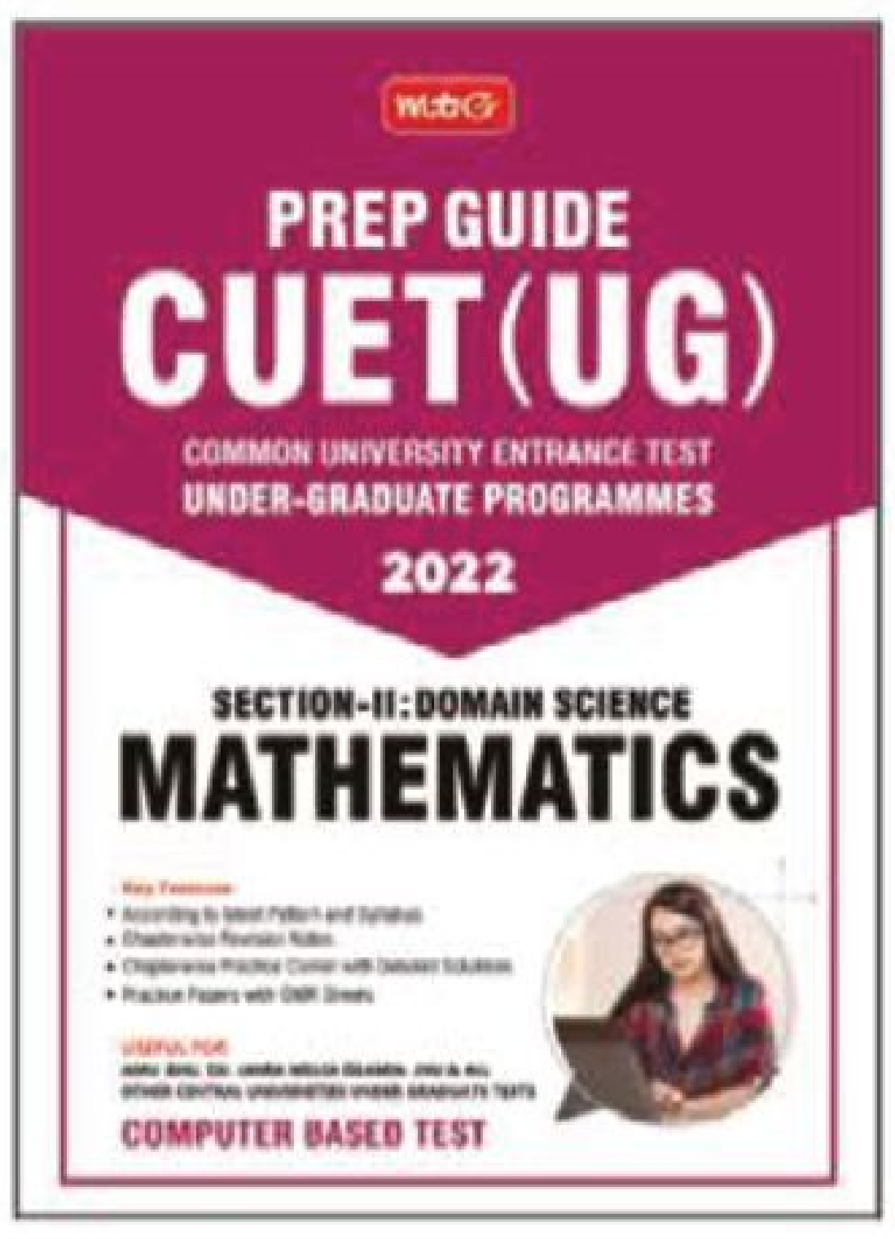
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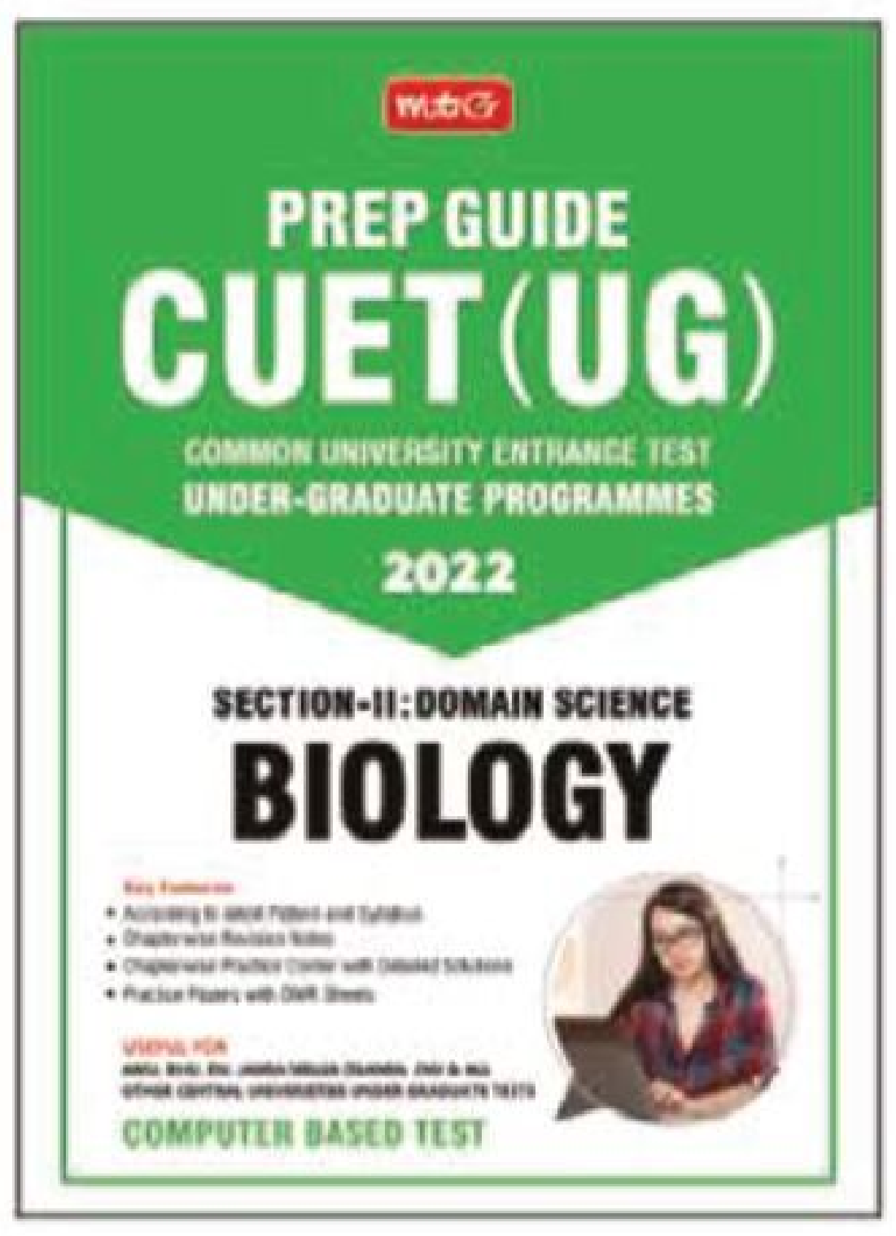
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During this journey, you will inevitably encounter more doubts than solutions; but don't be shy. Ask your educators to resolve them. Make a 'doubt copy' and collect all your queries in it, along with solutions. You can use this to revise before the main exam! Never let your doubts go unresolved.

JEE is not just another exam, nor is it a destination. It is a journey! The vital skills you will learn from this phase, like the value of hard work, perseverance and patience, will serve you in your future.

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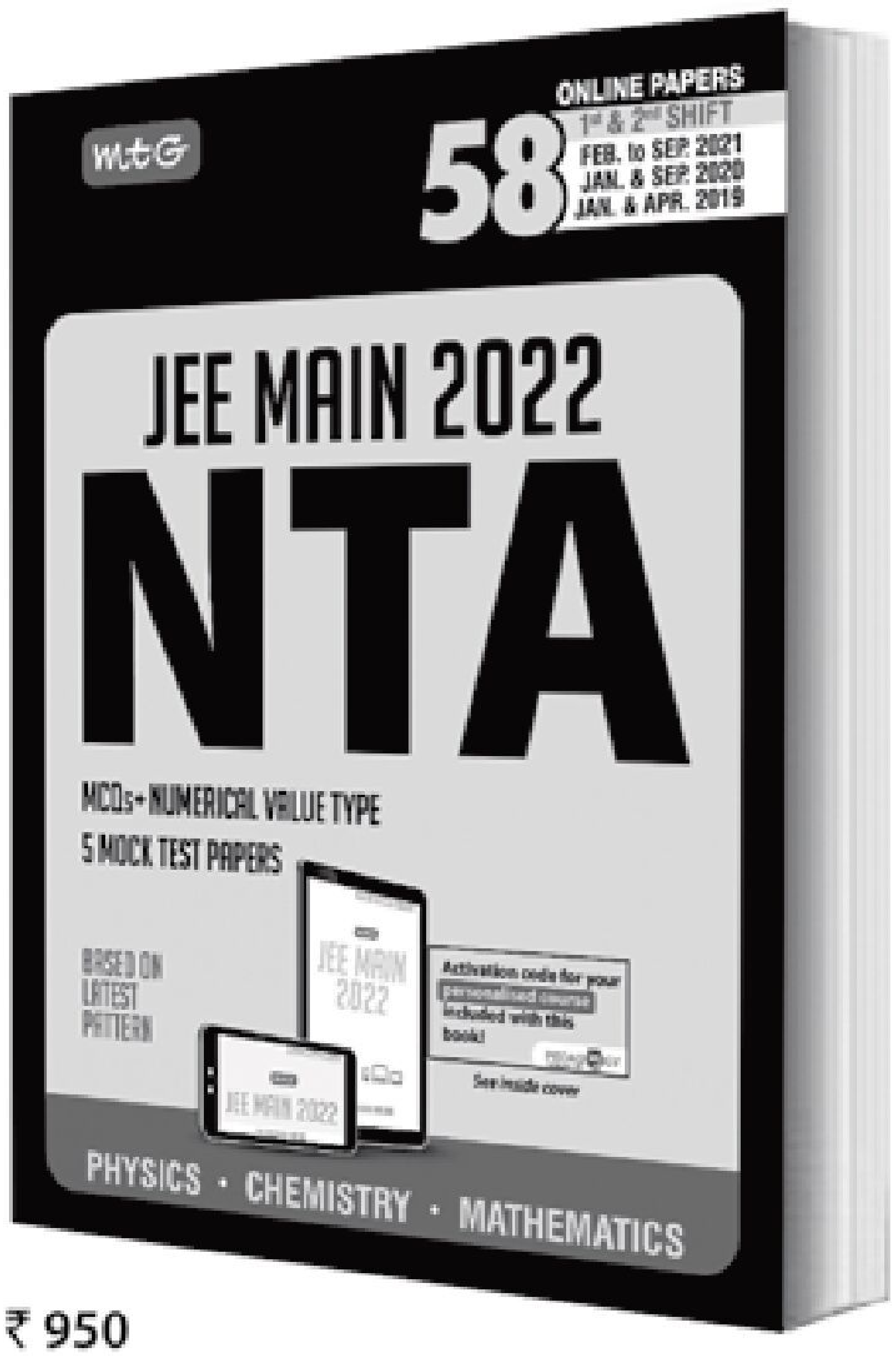
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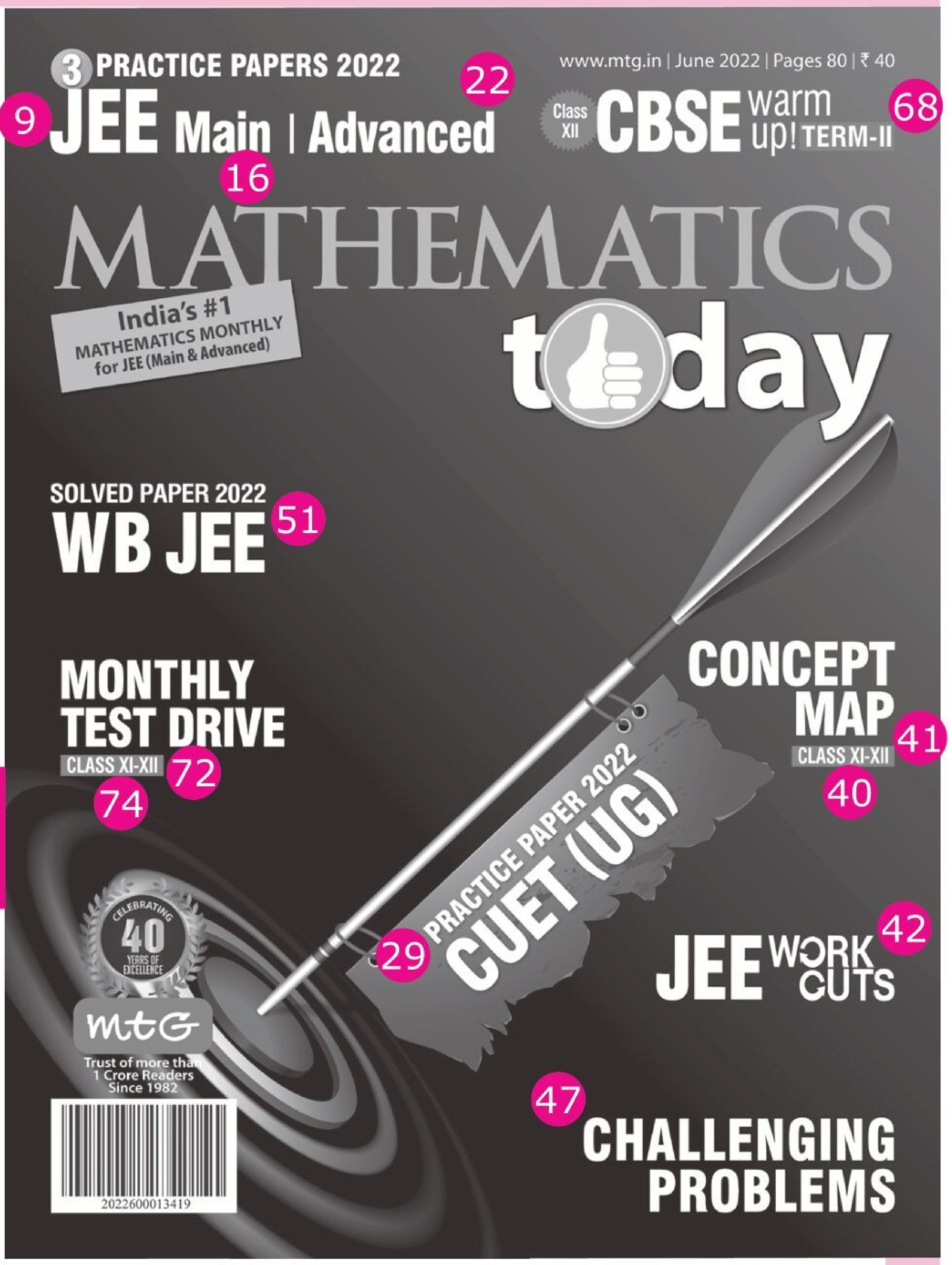
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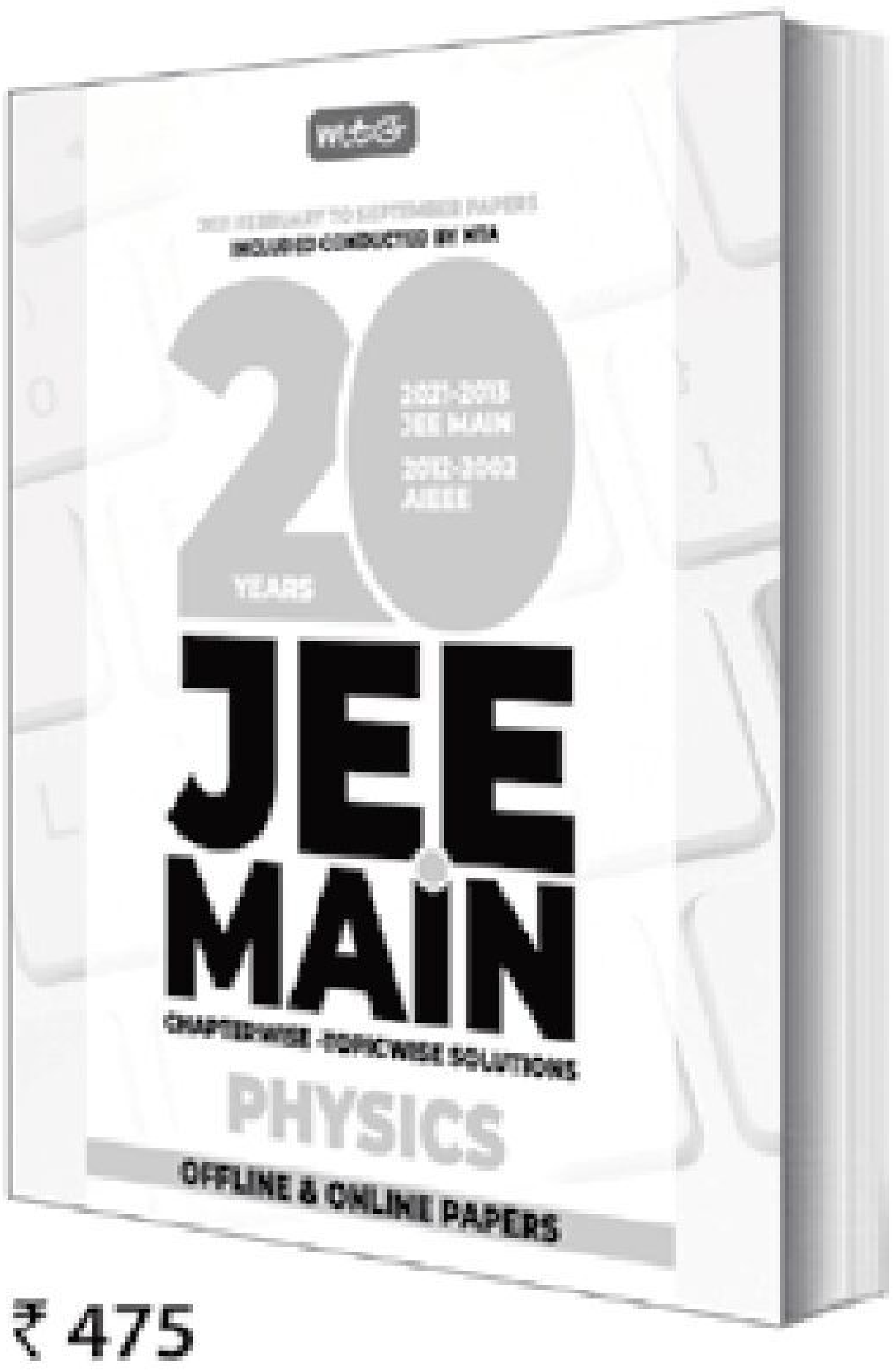
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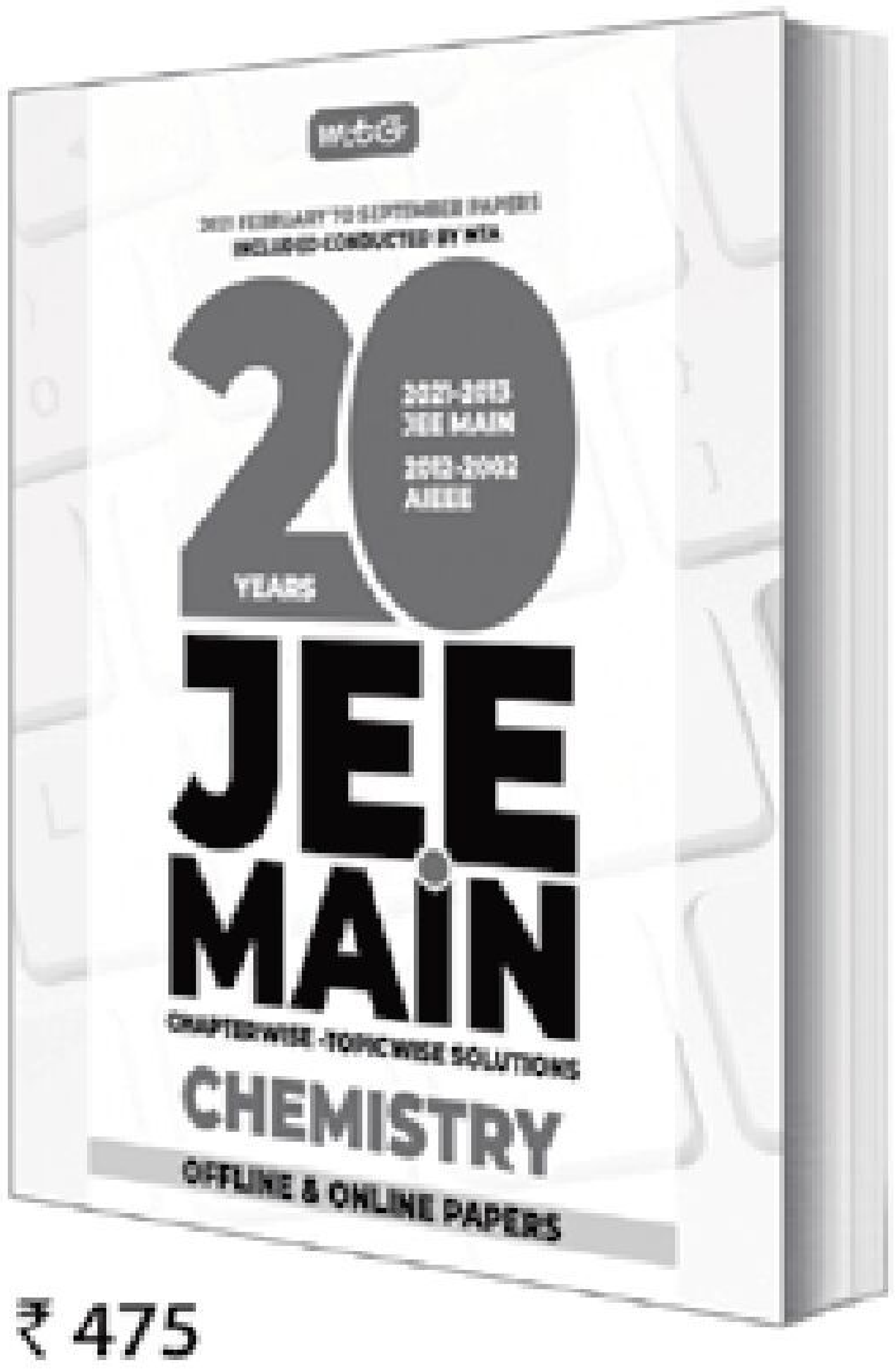
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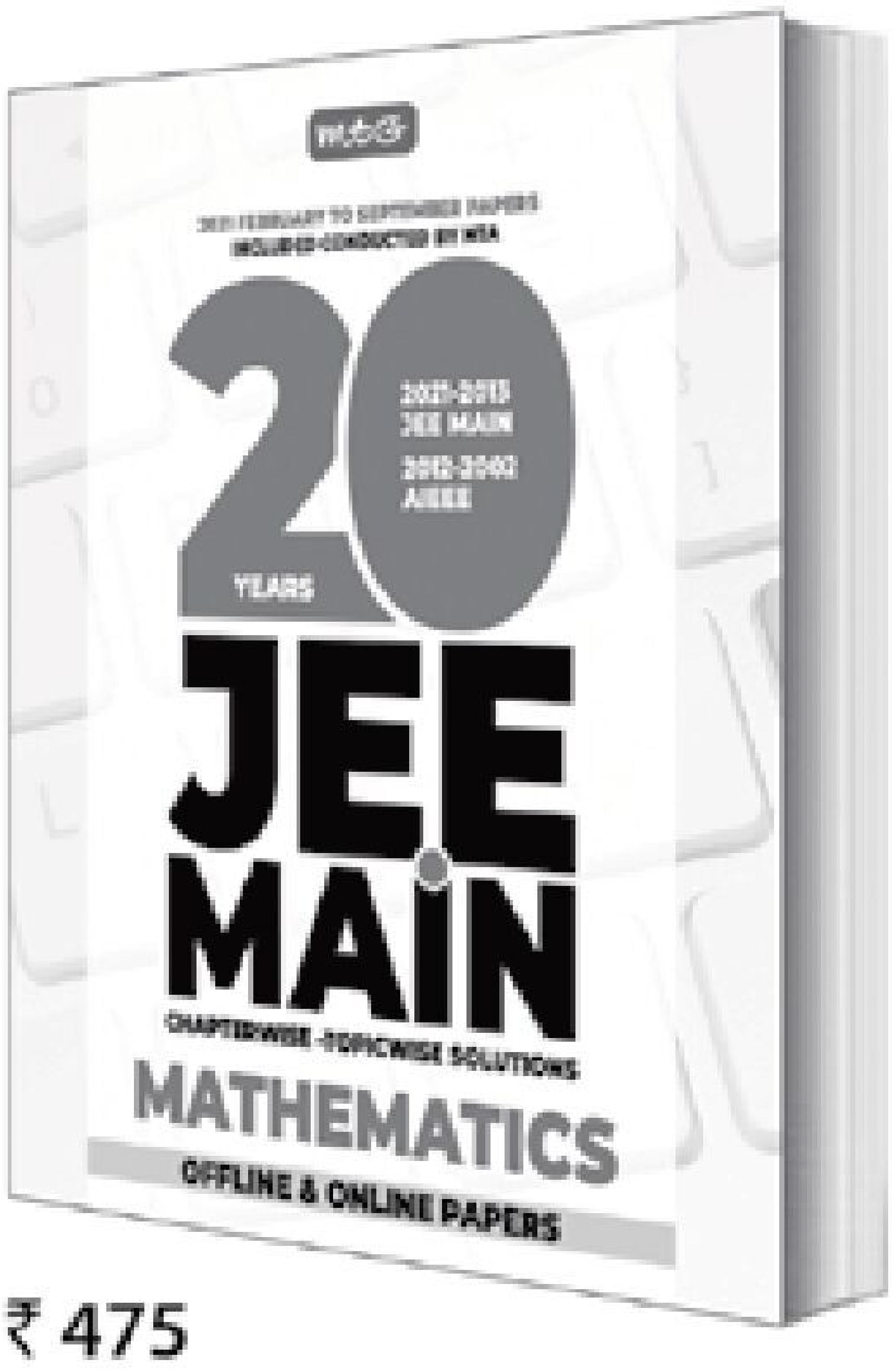
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JEE MAIN

Exam Dates
Session-1
20th to 29th June
Session-2
21st to 30th July

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. If $x = (2+\sqrt{3})^n$, $n \in N$ and $f = x - [x]$, then $\dfrac{f^2}{1-f}$ is

(a) an irrational number

(b) a non integer rational number

(c) an odd number

(d) an even number

2. If $f\left(x+\dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2}$, $x \neq 0$, then $f(x)$ is equal to

(a) x^2 (b) $x^2 - 1$ (c) $x^2 - 2$ (d) $x^2 + 1$

3. The domain of the function

$$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x), \text{ is}$$

(a) $(-1, 0) \cup (1, 2)$

(b) $(1, 2) \cup (2, \infty)$

(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(d) $(1, 2)$.

4. If $y = 2 + |x| - |x-1| - |x+1|$, then

$$y'\left(-\frac{1}{2}\right) + y'\left(\frac{1}{2}\right) + y'\left(\frac{3}{2}\right) + y'\left(\frac{5}{2}\right) =$$

(a) 0 (b) 1 (c) -1 (d) -2

5. If $\dfrac{{}^nP_{r-1}}{a} = \dfrac{{}^nP_r}{b} = \dfrac{{}^nP_{r+1}}{c}$, then

(a) $b^2 = a(b+c)$ (b) $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 1$

(c) $a^2 = c(a+b)$ (d) $abc = 1$

6. The function $f(x) = 1 - x^3 - x^5$ is decreasing for

(a) $1 \le x \le 5$ (b) $x \le 1$

(c) $x \ge 1$ (d) all values of x

7. Solve for y : $2y^2 + 6y + \dfrac{17}{2} = 0$

(a) $\dfrac{3}{2} \pm i\sqrt{2}$ (b) $-\dfrac{3}{2} \pm i\sqrt{2}$

(c) $\dfrac{1}{2} \pm i\sqrt{2}$ (d) $-\dfrac{1}{2} \pm i\sqrt{2}$

8. If $A = \begin{pmatrix} 2 & 3-i & -i \\ 3+i & -5 & 7+i \\ i & 7-i & e \end{pmatrix}$, then A is

(a) symmetric (b) hermitian

(c) skew hermitian (d) none of these

9. If x, $2y$, $3z$ are in A.P. where the distinct numbers x, y, z are in G.P., the common ratio of the G.P. is

(a) 1 (b) 1/2 (c) 1/3 (d) 2/5

10. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is

(a) 9 (b) 4 (c) 3 (d) 2

11. Evaluate : $I = \displaystyle\int_0^{\pi/2} \frac{1}{a^2\sin^2 x + b^2\cos^2 x}\,dx$

(a) $\dfrac{\pi a}{4b}$ (b) $\dfrac{\pi a}{2b}$ (c) $\dfrac{\pi b}{4a}$ (d) $\dfrac{\pi}{2ab}$

12. If $\displaystyle\lim_{x \to -1} \frac{x^3 + ax^2 + bx + c}{(x+1)^3}$ is a finite non-zero number, then $a + b + c =$

(a) 3 (b) 5 (c) 7 (d) 9

13. If $(1+x)\dfrac{dy}{dx} - xy = 1$, $y(0) = -1$, then $y(1) =$

(a) $-\dfrac{1}{2}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{\sqrt{e}}$ (d) $\sqrt{e}$

14. If $a \neq p$, $b \neq q$, $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\dfrac{p}{p-a} + \dfrac{q}{q-b} + \dfrac{r}{r-c}$ is

(a) 1 (b) -1 (c) 0 (d) 2

15. Let $A = \{1, 3, 5, 7, 9, 11\}$, then number of subsets of A is equal to

(a) 6^2 (b) 2^6 (c) 6 (d) 3^6

16. The distance of the point $A(-2, 3, 1)$ from the line PQ through $P(-3, 5, 2)$ which makes equal angles with the axes is

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{\frac{14}{3}}$ (c) $\frac{16}{\sqrt{3}}$ (d) $\frac{5}{\sqrt{3}}$

17. Tangents are drawn from the point $P(-2, -1)$ to the parabola $y^2 = 4x$. AB is the chord of contact. The normals at A and B meet at C. The area of triangle ABC is

(a) 8 (b) 4 (c) 5 (d) 6

18. Two events A and B will be independent, if

(a) A and B are mutually exclusive
(b) $P(A' \cap B') = [1 - P(A)]\,[1 - P(B)]$
(c) $P(A) = P(B)$
(d) $P(A) + P(B) = 1$

19. The four lines $x + 2y = 3$, $3x + 4y = 7$, $2x + 3y = 4$, $4x + 5y = 6$

(a) are all concurrent
(b) are the sides of a noncyclic quadrilateral
(c) form a cyclic quadrilateral
(d) none of these

20. If x_1, x_2, x_3, x_4 are the roots of $x^4 - x^3 \sin 2\alpha + x^2 \cos 2\alpha - x\cos\alpha - \sin\alpha = 0$, then $\sum_{i=1}^{4} \tan^{-1} x_i =$

(a) α (b) $\frac{\pi}{2} - \alpha$ (c) $-\alpha$ (d) $\pi - \alpha$

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. If the equation $\cot^4 x - 2\,\text{cosec}^2\, x + a^2 = 0$ has atleast one solution, then sum of all possible integral values of 'a' is equal to ______.

22. Let A be a 3×3 diagonal matrix which commutes with every 3×3 matrix. If $\det A = 8$, then tr A is

23. Given $f(x) = 2x^3 - 3(\lambda + 1)x^2 + 6(2\lambda + 1)x + \mu$, where $\mu \in R$ and $\lambda \in (-10, 10)$. If $f(x)$ has a positive point of local maxima, then number of possible integral values of λ is ______.

24. If $\frac{N}{1!18!} = \frac{1}{2!17!} + \frac{1}{3!\,16!} + \frac{1}{4!\,15!} + \frac{1}{5!\,14!} + \frac{1}{6!\,13!} + \frac{1}{7!\,12!} + \frac{1}{8!\,11!} + \frac{1}{9!\,10!}$, then the greatest integer less than $\frac{N}{10^4}$ is

25. The area enclosed by the curve $xy^2 = 4(2 - x)$ and y-axis is λ sq units, then the value of $5050 \cos(1824\lambda)$ must be _______ .

26. If $N = \sum_{1 \le r < s \le 20} (r - s)^2$, where $r, s \in \{1, 2, 3, \ldots\ldots, 20\}$, then the number of prime factors of N is

27. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + x_1\hat{j} + x_2\hat{k}$ are linearly dependent vectors and $|\vec{c}| = 2,$, then the value of $729x_1^4 + 343x_1^2x_2^2 + 81x_2^2 + 15$ must be _____.

28. The distance of the point $\hat{i} + 2\hat{j} + 3\hat{k}$ from the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$ measured parallel to the vector $2\hat{i} + 3\hat{j} - 6\hat{k}$ must be ______.

29. If the normal to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ at the point 'θ' cuts the curve again at the point '2θ', then $4\sec^2\theta$ is

30. If $a \in [-6, 12]$, then the probability that graph of $y = -x^2 + 2(a + 4)x - (3a + 40)$ is strictly below x-axis, is $\frac{m}{n}$, then $m + n$ is _______.

SOLUTIONS

1. (d): $\left(2+\sqrt{3}\right)^n = [x] + f$.

Let $\left(2-\sqrt{3}\right)^n = f', 0 < f' < 1$

$$[x] + f + f' = \left(2+\sqrt{3}\right)^n + \left(2-\sqrt{3}\right)^n$$

$$= 2\left[\binom{n}{0}2^n + \binom{n}{2}2^{n-2}\cdot 3 + \ldots\right] \text{an even integer}$$

$\therefore$ $[x]$ is an odd integer, $f' = 1 - f$.

But $([x] + f)f' = \left(2+\sqrt{3}\right)^n\left(2-\sqrt{3}\right)^n = 1$

$$\therefore [x] = \frac{1}{f'} - f = \frac{1}{1-f} - f = \frac{1 - f + f^2}{1-f}$$

$$\therefore \frac{f^2}{1-f} = [x] - 1, \text{ an even integer.}$$

2. (c) : $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$\therefore$ $f(x) = x^2 - 2$.

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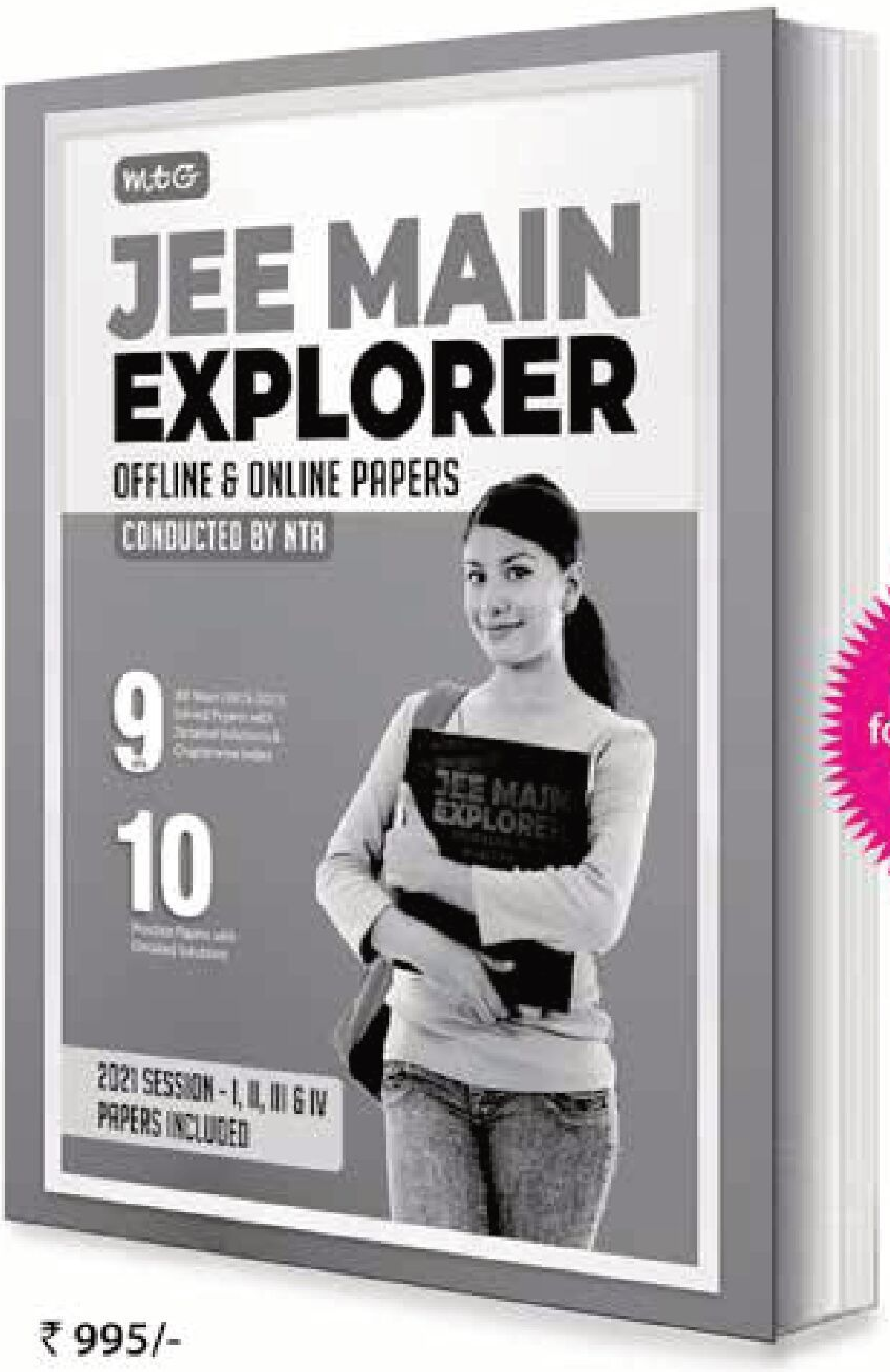
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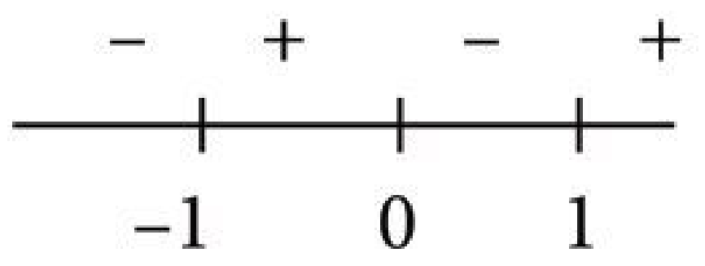
3. (c) : Let $g(x) = \dfrac{3}{4-x^2}$. Then, $4 - x^2 \neq 0 \Rightarrow x \neq \pm 2$

$\therefore\ D(g(x)) = R - \{-2, 2\}$

Also, let $h(x) = \log_{10}(x^3 - x)$ Then, $x^3 - x > 0$

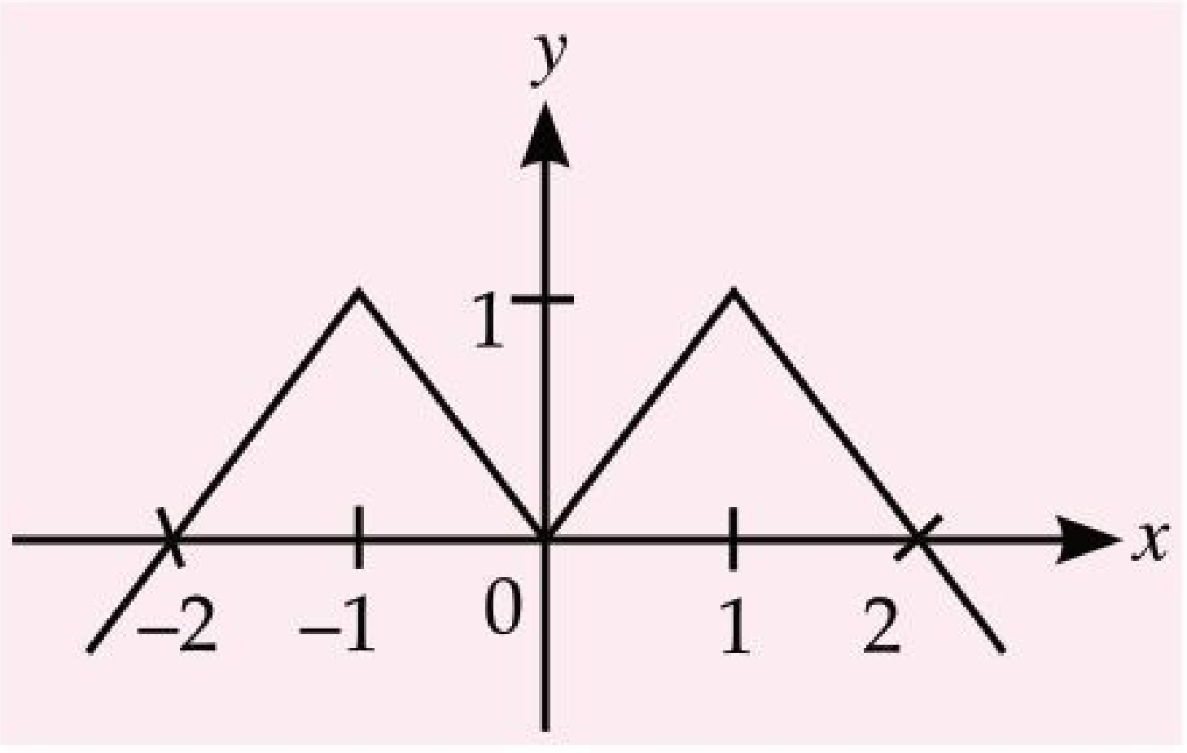
$\Rightarrow x(x + 1)(x - 1) > 0$



$\therefore\ x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of $f(x)$ is $(-1, 0) \cup (1, 2) \cup (2, \infty)$.

4. (d): When $x < -1$



$\Rightarrow f(x) = 2 - x + x - 1 + x + 1$

$= x + 2$

When $-1 < x < 0$

$\Rightarrow f(x) = 2 - x + x - 1 - x - 1$

$= -x$

When $0 < x < 1$

$\Rightarrow f(x) = 2 + x + x - 1 - x - 1 = x$

When $1 \leq x \Rightarrow f(x) = 2 + x - x + 1 - x - 1 = 2 - x$

$$y'\left(-\frac{1}{2}\right) + y'\left(\frac{1}{2}\right) + y'\left(\frac{3}{2}\right) + y'\left(\frac{5}{2}\right)$$

$= -1 + 1 - 1 - 1 = -2.$

5. (a) : $\dfrac{1}{a}\dfrac{n!}{(n-r+1)!} = \dfrac{1}{b}\dfrac{n!}{(n-r)!} = \dfrac{1}{c}\dfrac{n!}{(n-r-1)!}$

$\Rightarrow \dfrac{b}{a} = n - r + 1$ ...(i) and $\dfrac{c}{b} = n - r$ ...(ii)

From (i) and (ii), we get

$\dfrac{b}{a} = \dfrac{c}{b} + 1 \Rightarrow b^2 = a(b + c).$

6. (d): Given, $f(x) = 1 - x^3 - x^5$

Differentiating w.r.t. x, we get

$f'(x) = -3x^2 - 5x^4$

$\Rightarrow f'(x) = -(3x^2 + 5x^4) \Rightarrow f'(x) < 0$ for all values of x.

7. (b): We have, $2y^2 + 6y + \dfrac{17}{2} = 0$

$\Rightarrow y^2 + 3y + \dfrac{17}{4} = 0$

On adding and subtracting $\left(\dfrac{3}{2}\right)^2$, we get

$$y^2 + 2(y)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{17}{4} = 0$$

$\Rightarrow \left(y + \dfrac{3}{2}\right)^2 + 2 = 0 \Rightarrow \left(y + \dfrac{3}{2}\right)^2 - \left(i\sqrt{2}\right)^2 = 0$

$\Rightarrow \left(y + \dfrac{3}{2} + i\sqrt{2}\right)\left(y + \dfrac{3}{2} - i\sqrt{2}\right) = 0$

Either $y + \dfrac{3}{2} + i\sqrt{2} = 0$ or $y + \dfrac{3}{2} - i\sqrt{2} = 0$

$\Rightarrow y = -\dfrac{3}{2} - i\sqrt{2}$ or $y = -\dfrac{3}{2} + i\sqrt{2}$

Hence, roots of the given equation are $-\dfrac{3}{2} - i\sqrt{2}$ and $-\dfrac{3}{2} + i\sqrt{2}$.

8. (b): $A = \begin{bmatrix} 2 & 3-i & -i \\ 3+i & -5 & 7+i \\ i & 7-i & e \end{bmatrix}$

$$\overline{A} = \begin{bmatrix} 2 & 3+i & i \\ 3-i & -5 & 7-i \\ -i & 7+i & e \end{bmatrix}$$

$$(\overline{A})' = \begin{bmatrix} 2 & 3-i & -i \\ 3+i & -5 & 7+i \\ i & 7-i & e \end{bmatrix} = A$$

So, A is hermitian matrix.

9. (c) : x, y, z are in G.P. $\Rightarrow x = \dfrac{y}{r}$, $z = yr$ and $x, 2y, 3z$ are in A.P.

i.e., $\dfrac{y}{r}, 2y, 3yr$ are in A.P.

$\Rightarrow \dfrac{y}{r} + 3yr = 4y \Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow r = 1, \dfrac{1}{3}$. But $r \neq 1 \ \therefore\ r = \dfrac{1}{3}$.

10. (d): $\Sigma x_i - 45 = 9 \Rightarrow \Sigma x_i = 54$

Now, $\Sigma x_i^2 - 10 \times 54 + 25 \times 9 = 45 \Rightarrow \Sigma x_i^2 = 360$

$$\therefore\ \sigma = \sqrt{\frac{360}{9} - \left(\frac{54}{9}\right)^2} = 2$$

11. (d): Dividing numerator and denominator by $\cos^2 x$, we get

$$I = \int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 \tan^2 x + b^2}$$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

Now, $x = 0 \Rightarrow t = 0$ and $x = \dfrac{\pi}{2} \Rightarrow t = \infty$

$$\therefore\ I = \int_0^{\infty} \frac{dt}{a^2 t^2 + b^2} = \frac{1}{a^2}\int_0^{\infty} \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

$$= \frac{1}{a^2} \times \frac{1}{\left(\frac{b}{a}\right)}\left[\tan^{-1}\frac{t}{(b/a)}\right]_0^{\infty} = \frac{1}{ab} \times \frac{\pi}{2} = \frac{\pi}{2ab}$$

12. (c) : Let $x + 1 = t$.

$$L = \lim_{t\to 0}\frac{(t-1)^3 + a(t-1)^2 + b(t-1) + c}{t^3}$$

$$= \lim_{t\to 0}\frac{t^3 + (a-3)t^2 + (3-2a+b)t - 1 + a - b + c}{t^3}$$

$$= \lim_{t\to 0}\left[1 + \frac{(a-3)}{t} + \frac{3-2a+b}{t^2} + \frac{(-1+a-b+c)}{t^3}\right]$$

$\Rightarrow$ $a = 3$, $3 - 2a + b = 0$, $-1 + a - b + c = 0$

$\Rightarrow$ $a = 3$, $b = 3$, $c = 1 \Rightarrow a + b + c = 7$.

13. (a) : $\dfrac{dy}{dx} - \dfrac{xy}{1+x} = \dfrac{1}{x+1}$

It is a linear differential equation.

$\therefore$ Integrating factor (I.F.) $= e^{\int \frac{-x}{1+x}dx}$

$$= e^{\int\left(\frac{1}{1+x}-1\right)dx} = e^{[\ln(1+x) - x]}$$

$$= (1+x)e^{-x}$$

$\therefore$ Solution is $y(1+x)e^{-x} = \int e^{-x}dx$

$\Rightarrow$ $y(1+x)e^{-x} = -e^{-x} + c$

$\Rightarrow$ $y(1+x) = -1 + ce^{x}$

$\because$ $x = 0, y = -1 \Rightarrow c = 0 \Rightarrow y = -\dfrac{1}{1+x} \Rightarrow y(1) = -\dfrac{1}{2}$.

14. (d): Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ gives

$$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = 0$$

Applying $C_1 \to \dfrac{C_1}{p-a}$, $C_2 \to \dfrac{C_2}{q-b}$, $C_3 \to \dfrac{C_3}{r-c}$, we get

$$\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Expanding by R_1, we get $\dfrac{p}{p-a} + \dfrac{b}{q-b} + \dfrac{c}{r-c} = 0$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} - 1 + \frac{r}{r-c} - 1 = 0$$

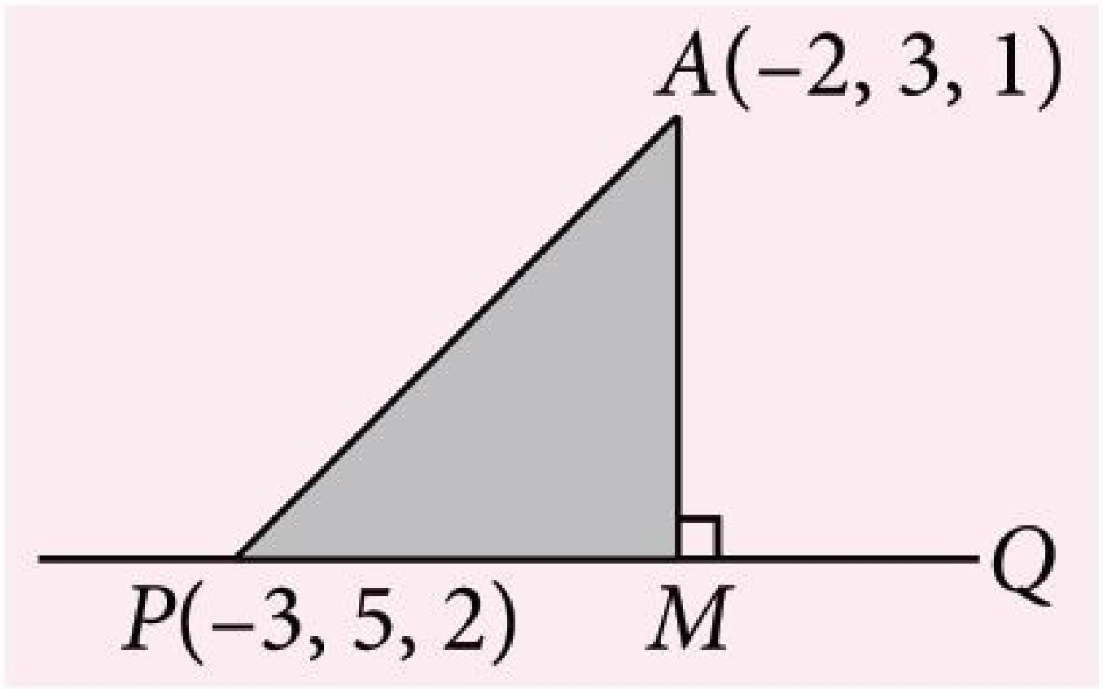
$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2.$$

15. (b): The number of subsets of $A = 2^6$.

16. (b): Here, $\alpha = \beta = \gamma$

$\because$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore$ $\cos\alpha = \dfrac{1}{\sqrt{3}}$



DC's of PQ are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

PM = Projection of AP on PQ

$$= \left|(-2+3)\frac{1}{\sqrt{3}} + (3-5)\cdot\frac{1}{\sqrt{3}} + (1-2)\cdot\frac{1}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$$

and $AP = \sqrt{(-2+3)^2 + (3-5)^2 + (1-2)^2} = \sqrt{6}$

$$AM = \sqrt{(AP)^2 - (PM)^2} = \sqrt{6 - \frac{4}{3}} = \sqrt{\frac{14}{3}}$$

17. (d): Let the points A and B be t_1 and t_2.

The tangents at A and B meet at

$P(t_1t_2, t_1 + t_2) = (-2, -1)$ $\therefore$ $t_1 = 1$, $t_2 = -2$

The normal at t is $xt + y = t^3 + 2t$.

The normals are $x + y = 3$, $-2x + y = -12$

Solving, $C \equiv (5, -2)$, But $A \equiv (1, 2)$

and $B \equiv (4, -4)$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & -4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = 6.$$

18. (b): Two events A and B are independent, if

$P(A \cap B) = P(A)\,P(B)$

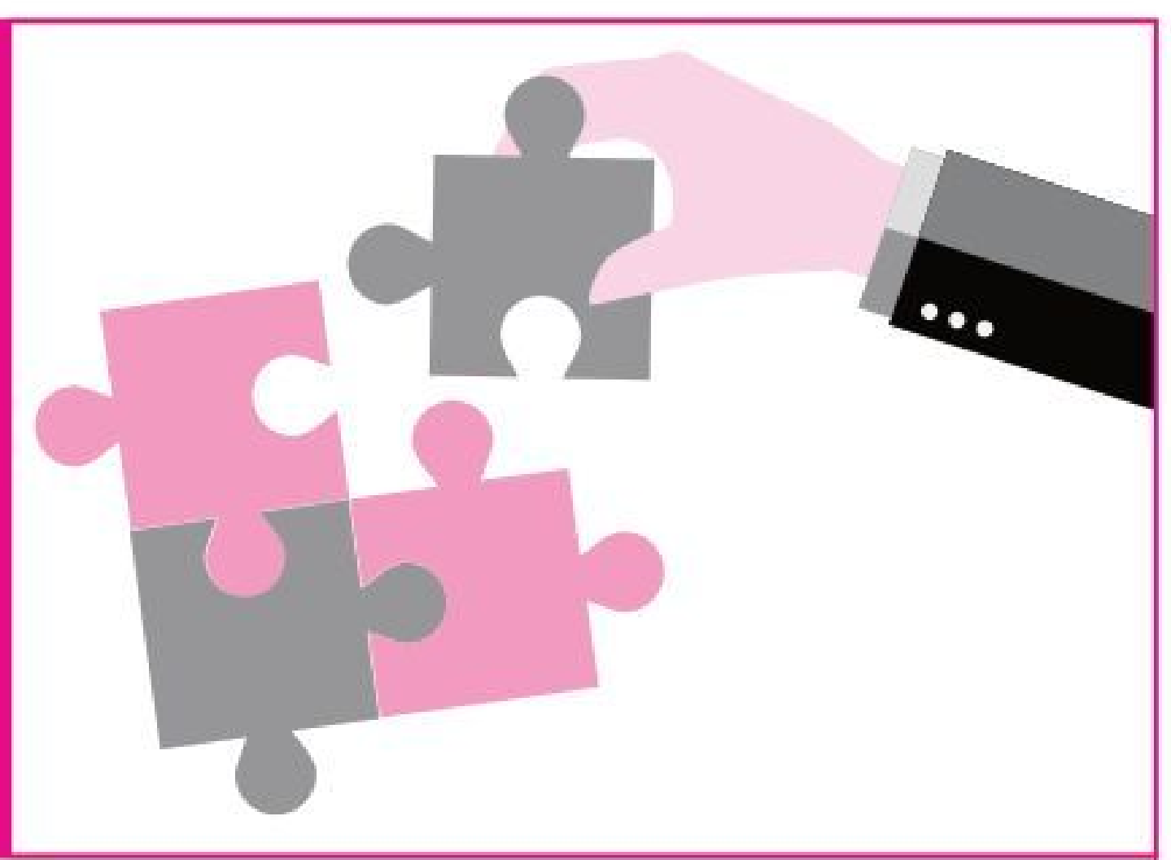
Now, consider $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - P(A) - P(B) + P(A)\,P(B)$$

$$= [1 - P(A)][1 - P(B)]$$

PUZZLE CORNER



ANSWER - MAY 2022

2– 3	5	24× 1	4	6	2÷ 2
3÷ 6	2	3– 5	17+ 3	1– 4	1
6× 1	3	2	6	5	24× 4
2	3– 1	4 4	5	3	6
1– 5	4	4– 6	2	1– 1	2– 3
4	2÷ 6	3	1 1	2	5

19. (d): $x + 2y = 3$, $3x + 4y = 7$ meet at $(1, 1)$ which does not lie on the other lines.

$\therefore$ The lines are not concurrent.

The three lines $x + 2y - 3 = 0$, $2x + 3y - 4 = 0$, $4x + 5y - 6 = 0$ are concurrent since

$$\begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & -4 \\ 4 & 5 & -6 \end{vmatrix} = -18 + 20 - 2(-12 + 16) - 3(-2) = 0$$

They are concurrent at $(-1, 2)$ and the other line $3x + 4y - 7 = 0$ does not pass through it.

20. (b): $\sum x_i = \sin 2\alpha$, $\sum_{i<j} x_i x_j = \cos 2\alpha$,

$\sum_{i<j<k} x_i x_j x_k = \cos \alpha$, $x_1 x_2 x_3 x_4 = -\sin \alpha$

$$\sum_{i=1}^{4} \tan^{-1} x_i = \tan^{-1} \frac{\Sigma x_i - \Sigma x_i x_j x_k}{1 - \Sigma x_i x_j + x_1 x_2 x_3 x_4}$$

$$= \tan^{-1}\left(\frac{\sin 2\alpha - \cos \alpha}{1 - \cos 2\alpha - \sin \alpha}\right)$$

$$= \tan^{-1}\left(\frac{\cos\alpha(2\sin\alpha - 1)}{\sin\alpha\,(2\sin\alpha - 1)}\right)$$

$$= \tan^{-1} \cot\alpha = \tan^{-1} \tan\left(\frac{\pi}{2} - \alpha\right) = \frac{\pi}{2} - \alpha.$$

21. (0): $\cot^4 x - 2(1 + \cot^2 x) + a^2 = 0$

$\Rightarrow$ $\cot^4 x - 2\cot^2 x + a^2 - 2 = 0$

$\Rightarrow$ $(\cot^2 x - 1)^2 = 3 - a^2$

The given equation have atleast one solution of

$3 - a^2 \geq 0$

$\Rightarrow$ $a^2 - 3 \leq 0$

$\Rightarrow$ $a \in [-\sqrt{3}, \sqrt{3}]$

Thus integral values are $-1, 0, 1$ $\therefore$ Sum = 0

22. (6): $\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}$

$\therefore$ $\alpha = \beta = \gamma$

$\therefore$ $A = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix}$, $|A| = \alpha^3 = 8 \Rightarrow \alpha = 2$

$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, tr $A = 2 + 2 + 2 = 6$.

23. (3) : Given, $f(x) = 2x^3 - 3(\lambda + 1)x^2 + 6(2\lambda + 1)x + \mu$

$\therefore$ $f'(x) = 6[x^2 - (\lambda + 1)x + (2\lambda + 1)]$

$\because$ $f(x)$ has a positive point of local maxima, therefore, the equation $f'(x) = 0$ must have both positive and distinct roots.

$\therefore$ $D > 0 \Rightarrow (\lambda + 1)^2 - 4(2\lambda + 1) > 0$

$\Rightarrow$ $(\lambda - 3)^2 > 12 \Rightarrow |\lambda - 3| > \sqrt{12}$

$\Rightarrow$ $\lambda \in (-\infty, 3 - 2\sqrt{3}) \cup (3 + 2\sqrt{3}, \infty)$ ...(i)

Also, $\frac{\lambda + 1}{2} > 0 \Rightarrow \lambda > -1$ ...(ii)

and $f'(0) > 0 \Rightarrow \lambda > -\frac{1}{2}$ ...(iii)

and $\lambda \in (-10, 10)$ ...(iv)

From intersection of (i), (ii), (iii) and (iv) the integral values of λ are 7, 8, 9. *i.e.*, 3 in number.

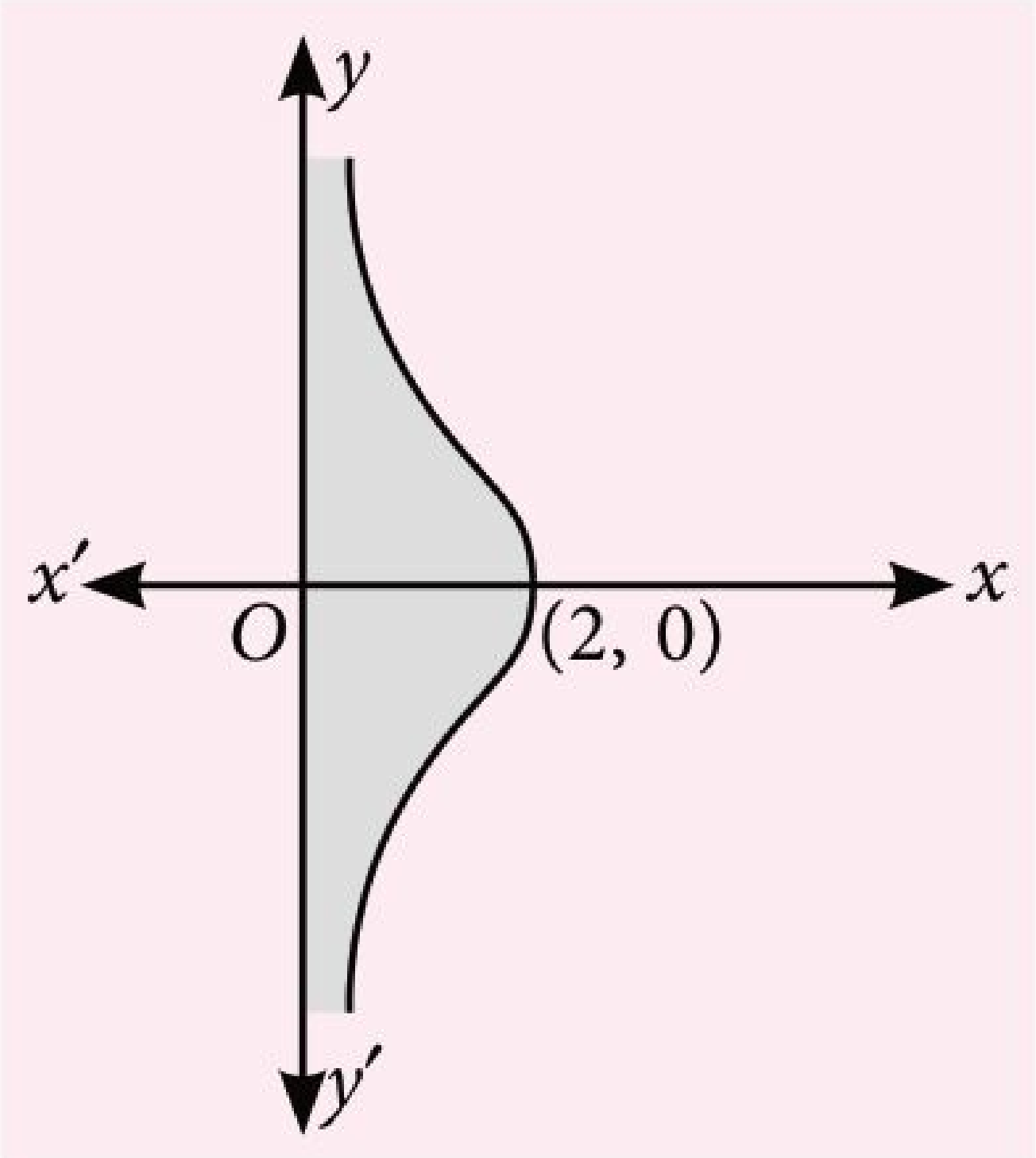
24. (1): $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n$

$\therefore$ $19N = \frac{2^{19} - 2 - 38}{2} = 2^{18} - 20$

$\frac{N}{10^4} = \frac{2^{16} - 5}{19 \times 2500} = \frac{65531}{47500}.$

25. (5050) : The curve $xy^2 = 4(2 - x)$ is symmetrical about x-axis and at $x \to 0$, $y \to \infty$

i.e., y-axis is an asymptote and cut off x-axis at $x = 2$



$\because$ $y^2 = \frac{4(2 - x)}{x} \geq 0$

$\Rightarrow$ $\left(\frac{2 - x}{x}\right) \geq 0$

$\Rightarrow$ $\left(\frac{x - 2}{x}\right) \leq 0$ $\therefore$ $x \in (0, 2]$

Then,

$$\lambda = 2\int_0^2 2\sqrt{\left(\frac{2 - x}{x}\right)}\,dx$$

Put $x = 2\sin^2\theta$

$\therefore$ $dx = 4\sin\theta\cos\theta\,d\theta$

$$\Rightarrow \lambda = 4\int_0^{\pi/2} \sqrt{\left(\frac{2 - 2\sin^2\theta}{2\sin^2\theta}\right)} \times 4\sin\theta\cos\theta\,d\theta$$

$$= 16\int_0^{\pi/2} \cos^2\theta\,d\theta = 16\frac{1}{2}\cdot\frac{\pi}{2} = 4\pi$$

$\therefore$ $5050\cos(1824\lambda) = 5050 \times 1 = 5050$

26. (4) : $N=(1-2)^2+(1-3)^2+\ldots\ldots.+(1-20)^2+(2-3)^2$ $+\ldots\ldots\ldots+(2-20)^2\ldots\ldots+(19-20)^2$

$$=19\,(1^2+2^2+\ldots+20^2)-2\sum_{1\le r<s\le 20} rs$$

$$=19\,(1^2+2^2+\ldots.+20^2)-(1+2+\ldots\ldots.+20)^2 + (1^2+2^2+\ldots..+20^2)$$

$$=20(1^2+2^2+\ldots+20^2)-(1+2+\ldots\ldots..+20)^2$$

$$=\frac{20\cdot 20\cdot 21\cdot 41}{6}-\frac{20^2\cdot 21^2}{4}$$

$=(574-441)\times 100=13300=2^2\cdot 5^2\cdot 7\cdot 19.$

27. (3698) : Clearly $\vec{a},\vec{b}$ and $\vec{c}$ must be coplanar

$\Rightarrow\ \vec{c}=\lambda\,\vec{a}+\mu\,\vec{b}$

$\Rightarrow\ \hat{i}+x_1\hat{j}+x_2\hat{k}=\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(4\hat{i}+3\hat{j}+4\hat{k})$

$\therefore\ 1=\lambda+4\mu,\ x_1=\lambda+3\mu$ and $x_2=\lambda+4\mu$

$\therefore\ x_2=1$ ...(i)

Also, given $|\vec{c}|=2$

$\Rightarrow\ \sqrt{1+x_1^2+x_2^2}=2\ \Rightarrow\ 2+x_1^2=4$ [Using (i)]

$\therefore\ x_1^2=2$

Consider, $729x_1^4+343x_1^2x_2^2+81x_2^2+15$

$=729(2)^2+343\,(2)\,(1)^2+81\,(1)^2+15$

$=2916+686+81+15=3698$

28. (7) : The distance of the point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n}=q$ measured in the direction of the unit vector $\hat{b}$ is given by

$$\left|\frac{q-\vec{a}\cdot\vec{n}}{\hat{b}\cdot\vec{n}}\right|$$

Here $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\vec{n}=\hat{i}+\hat{j}+\hat{k},\ q=5$ and

$$\hat{b}=\frac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{(2)^2+(3)^2+(-6)^2}}=\frac{2\hat{i}+3\hat{j}-6\hat{k}}{7}$$

$\therefore$ The required distance

$$=\left|\frac{5-(\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})}{\frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})}\right|=\left|\frac{5-(1+2+3)}{\frac{1}{7}(2+3-6)}\right|=7$$

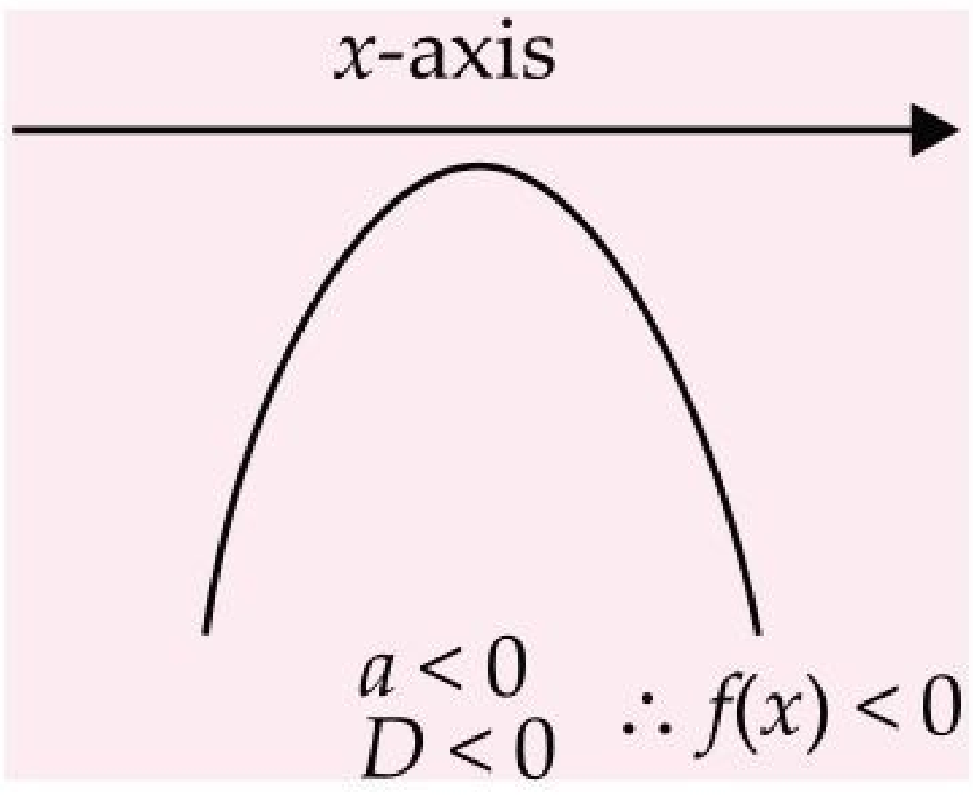
29. (9) : Normals at θ is $\dfrac{\sqrt{14}x}{\cos\theta}-\dfrac{\sqrt{5}y}{\sin\theta}=9$,

It passes through $(\sqrt{14}\cos 2\theta,\sqrt{5}\sin 2\theta)$

$$\therefore\ \frac{14\cos 2\theta}{\cos\theta}-\frac{5\sin 2\theta}{\sin\theta}=9$$

$\Rightarrow\ 18\cos^2\theta-9\cos\theta-14=0$

Solving, $\cos\theta=-\dfrac{2}{3},\dfrac{7}{6}\ \therefore\ 4\sec^2\theta=9.$



30. (3) : The total length of the interval

$=12-(-6)=18.$

If graph of $y=-x^2+2(a+4)x-(3a+40)$ is entirely below x-axis, the value of discriminant of the above quadratic expression must be negative.

$\therefore\ 4(a+4)^2-4(1)\,(3a+40)<0$

$\Rightarrow\ a^2+5a-24<0\ \Rightarrow\ (a+8)\,(a-3)<0$

$\Rightarrow\ -8<a<3$

but $a\in[-6,12]$

$\therefore\ -6<a<3$ for event to happen

$\therefore$ length of interval $=3-(-6)=9$

Hence, the required probability $=\dfrac{9}{18}=1/2$

◆◆

SAMURAI SUDOKU

ANSWER - MAY 2022

3	9	4	5	8	1	2	6	7				9	3	4	5	7	8	6	2	1
7	8	6	3	2	9	4	1	5				6	7	1	9	2	4	3	8	5
5	2	1	4	6	7	8	3	9				5	2	8	1	6	3	7	9	4
4	1	5	6	9	3	7	2	8				7	9	5	4	8	1	2	6	3
2	3	9	8	7	4	6	5	1				1	4	2	7	3	6	8	5	9
6	7	8	1	5	2	3	9	4				8	6	3	2	9	5	4	1	7
1	5	2	7	3	8	9	4	6	2	8	1	3	5	7	6	1	2	9	4	8
9	4	7	2	1	6	5	8	3	4	7	6	2	1	9	8	4	7	5	3	6
8	6	3	9	4	5	1	7	2	9	3	5	4	8	6	3	5	9	1	7	2
						4	2	1	8	5	9	7	6	3						
						8	3	9	1	6	7	5	2	4						
						6	5	7	3	4	2	1	9	8						
1	4	7	3	9	8	2	6	5	7	9	3	8	4	1	2	3	6	9	5	7
8	3	9	5	6	2	7	1	4	6	2	8	9	3	5	4	1	7	8	6	2
2	5	6	7	1	4	3	9	8	5	1	4	6	7	2	8	5	9	3	4	1
6	7	2	1	5	9	8	4	3				5	1	4	6	9	2	7	8	3
5	1	3	8	4	7	6	2	9				3	6	9	7	8	5	2	1	4
4	9	8	2	3	6	1	5	7				2	8	7	3	4	1	6	9	5
3	8	5	9	2	1	4	7	6				1	2	6	5	7	8	4	3	9
9	6	1	4	7	3	5	8	2				7	9	3	1	6	4	5	2	8
7	2	4	6	8	5	9	3	1				4	5	8	9	2	3	1	7	6

Exam Dates
Session-1
20th to 29th June
Session-2
21st to 30th July

JEE Main 2022

MOCK TEST PAPER

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfies the equation $x^2 - (a + d)x + k = 0$, then

(a) $k = bc$ (b) $k = ad$
(c) $k = a^2 + b^2 + c^2 + d^2$ (d) $k = ad - bc$

2. Let $n \in N$ and $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$

Then $a_0{}^2 - a_1{}^2 + a_2{}^2 - \ldots + a_{2n}{}^2 =$

(a) $a_n{}^2$ (b) 0 (c) a_n (d) $2a_n$

3. If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is

(a) 5 (b) 25 (c) 15 (d) 10

4. If x is a complex root of the equation
$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0,$$
then $x^{2007} + x^{-2007} =$

(a) 1 (b) – 1 (c) – 2 (d) 2

5. If A is the void set ϕ, then $P(A)$ has just one element ϕ *i.e.* $P(\phi) = \{\phi\}$. So number of elements of $P[P(P(\phi))]$ is

(a) 4 (b) 3 (c) 5 (d) 2^4

6. If $(1 + e^{x/y})dx + \left(1 - \frac{x}{y}\right)e^{x/y}dy = 0$, then

(a) $x - ye^{x/y} = c$ (b) $y - xe^{x/y} = c$
(c) $x + ye^{x/y} = c$ (d) $y + xe^{x/y} = c$

7. The number of solutions of $\cos 2\theta = \sin\theta$ in $(0, 2\pi)$ is

(a) 1 (b) 2 (c) 3 (d) 4

8. Area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$ is

(a) 1 sq. unit (b) $\frac{1}{2}$ sq. unit
(c) $\frac{1}{3}$ sq. unit (d) cannot be determined

9. The orthocentre of the triangle formed by the points (0, 0), (4, 0) and (3, 4) is

(a) (2, 0) (b) $\left(\frac{3}{2}, 2\right)$
(c) $\left(\frac{3}{4}, 3\right)$ (d) $\left(3, \frac{3}{4}\right)$

10. The length of minor axis of the ellipse with foci (± 2, 0) and eccentricity 1/3 is

(a) 4 (b) 3 (c) $8\sqrt{2}$ (d) $4\sqrt{2}$

11. If $f(x) = \frac{2 - \sqrt{x+4}}{\sin(2x)}$, $x \neq 0$ is continuous function at $x = 0$, then the value of $f(0)$ is

(a) –1/8 (b) 1/2 (c) 1 (d) 0

12. The number of ways in which seven persons can be arranged at a round table, if two particular persons may not sit together is

(a) 480 (b) 120 (c) 80 (d) 460

13. If $e^{f(x)} = \frac{10 + x}{10 - x}$, $x \in (-10, 10)$ and

$f(x) = kf\left(\frac{200x}{100 + x^2}\right)$, then $k =$

(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

14. A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

15. The range of the function $f(x) = |x|$ is

(a) $(0, \infty)$ (b) $(-\infty, 0)$

(c) $[0, \infty)$ (d) $(-\infty, 0]$

16. The normal to the curve $2y = 3 - x^2$ at $(1, 1)$ is

(a) $x + y = 0$ (b) $x + y + 1 = 0$

(c) $x - y + 1 = 0$ (d) $x - y = 0$

17. $\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right) =$

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$

(c) $\frac{16}{7}$ (d) none of these

18. The successive terms of an A.P. are $a_1, a_2, a_3, ...$. If $a_6 + a_9 + a_{12} + a_{15} = 20$, then $\sum_{r=1}^{20} a_r =$

(a) 75 (b) 100 (c) 120 (d) 150

19. A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is

(a) $\frac{4\hat{i}+3\hat{j}-\hat{k}}{\sqrt{26}}$ (b) $\frac{2\hat{i}-6\hat{j}-3\hat{k}}{7}$

(c) $\frac{3\hat{i}-2\hat{j}+6\hat{k}}{7}$ (d) $\frac{2\hat{i}-3\hat{j}-6\hat{k}}{7}$

20. If $x_1, x_2, ..., x_{18}$ are observations such that $\sum_{j=1}^{18}(x_j - 8) = 9$ and $\sum_{j=1}^{18}(x_j - 8)^2 = 45$, then the standard deviation of these observations is

(a) $\sqrt{\frac{81}{34}}$ (b) 5 (c) $\sqrt{5}$ (d) $\frac{3}{2}$

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. If the normals at the points A and B on $y^2 = 4x$ meet at the point $C(t^2, 2t)$, then the minimum value of t^2 is

22. If $\int \sin 4x \cdot e^{\tan^2 x} dx = a\cos^b x \cdot e^{\tan^2 x} + c$, then the value of a^{2b} must be equal to ______.

23. If $12^{4+2x^2} = (24\sqrt{3})^{3x^2-2}$, then $x^2 =$ ______.

24. Let $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors, $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c}$ are coplanar, then $\left|\frac{\alpha}{\beta}\right|$ is _____.

25. Ten straight lines are drawn in a plane such that no two lines are parallel and no three lines are concurrent. If N is the number of divisions of the plane made by these lines, then the last digit of N is

26. The absolute value of 'm' for which the straight line $3x - 2y + z + 3 = 0 = 4x - 3y + 4z + 1$ is parallel to the plane $2x - y + mz - 2 = 0$ is ______.

27. If the minimum value of a is $-k/2$, such that the function $f(x) = x^2 + ax + 5$, is increasing in $[1, 2]$. Then the value of k is ______.

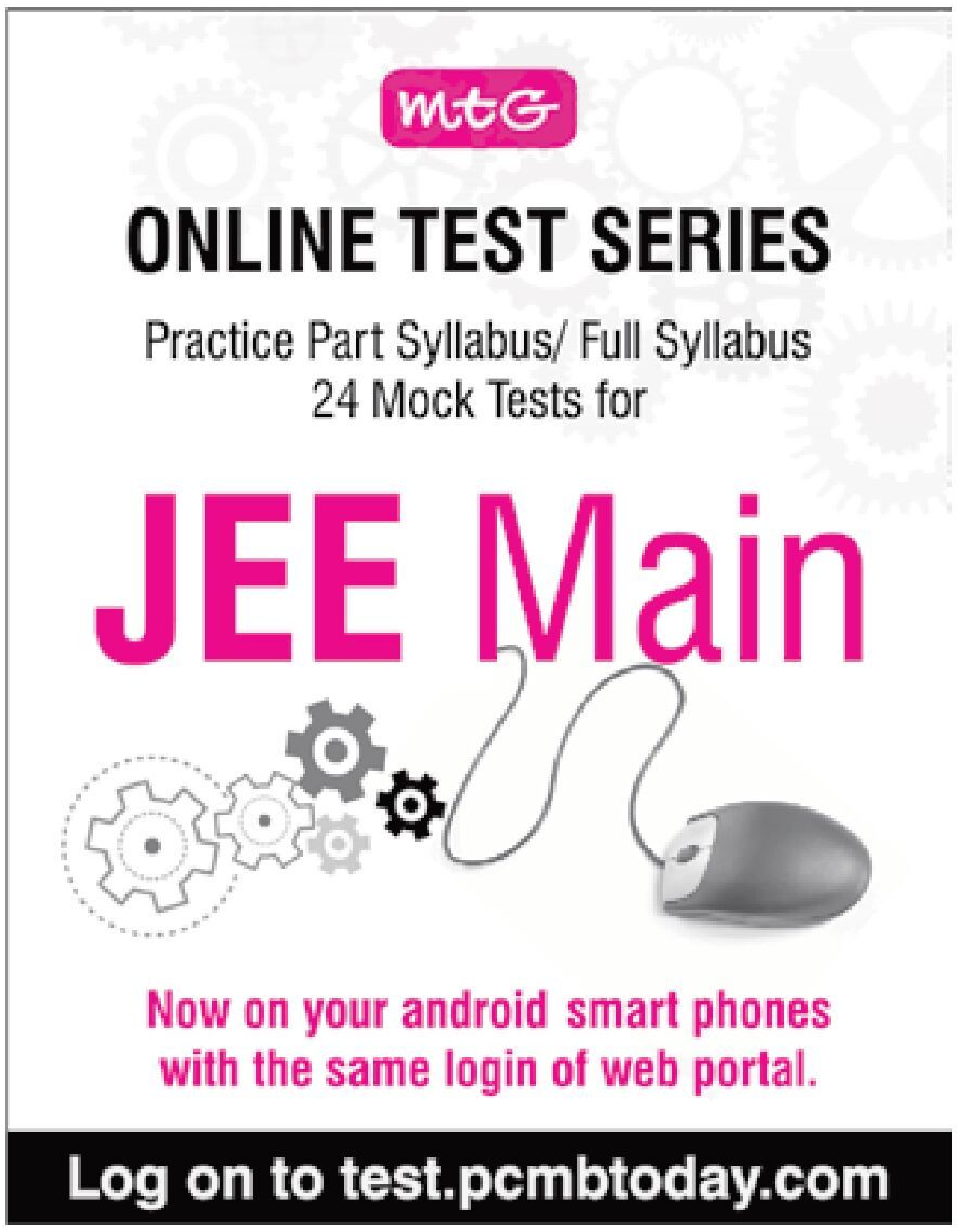
28. The sum of the last 4 digits of the natural number 3^{100} is ______.

29. Let X and Y be two events such that

$P(X) = \frac{1}{3}, P(Y) = \frac{4}{15}$ and $P(Y|X) = \frac{2}{5}$. Then

$P(X'|Y) =$ _______.

30. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then find the value of k.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SOLUTIONS

1. (d) : We have,

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$$\therefore \quad A^2-(a+d)A = \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix} = (bc-ad)I$$

As $A^2 - (a + d)A + kI = O$, we get

$(bc - ad)I + kI = O \Rightarrow k = ad - bc$

2. (c): We have, $a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{2n}x^{2n} = (1 + x + x^2)^n$

Replacing x by $-\frac{1}{x}$, we get

$$a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \ldots + \frac{a_{2n}}{x^{2n}} = \left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$$

Multiplying the above two, we get

$a_0^2 - a_1^2 + a_2^2 - \ldots + a_{2n}^2$ is the

constant term in $(1 + x + x^2)^n \left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$

$$= \frac{(1+x+x^2)^n(1-x+x^2)^n}{x^{2n}} = \frac{(1+x^2+x^4)^n}{x^{2n}}$$

$$= \frac{1}{x^{2n}}(a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n} + \ldots),$$

which is a_n.

3. (d) : We have, $y = f(x^2 + 2)$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = f'(x^2+2)(2x)$$

$$\therefore \left.\frac{dy}{dx}\right|_{\text{at } x=1} = 2f'(3) = 2 \times 5 = 10$$

4. (c) : Expanding the two determinants,

$(1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0$

$\Rightarrow x^3 + 1 = 0 \Rightarrow x = -1, -\omega, -\omega^2$

$\Rightarrow x^{2007} + x^{-2007} = -1 - 1 = -2.$

5. (a) : We have, $P(\phi) = \{\phi\}$

$\therefore \ P(P(\phi)) = \{\phi, \{\phi\}\}$

$\Rightarrow P[P(P(\phi))] = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Hence, $P[P(P(\phi))]$ consists of 4 elements *i.e.*, $n[P[P(P(\phi))]] = 4$

6. (c) : $\frac{dx}{dy} = \frac{\left(\frac{x}{y}-1\right)e^{x/y}}{1+e^{x/y}}$

Substitute $x = vy \Rightarrow \frac{dx}{dy} = \frac{ydv}{dy} + v$

Now, given equation becomes

$$\frac{ydv}{dy} + v = \frac{(v-1)e^v}{1+e^v}$$

$$\Rightarrow \frac{ydv}{dy} = \frac{(v-1)e^v}{1+e^v} - v = \frac{-(v+e^v)}{1+e^v}$$

$$\Rightarrow \int\frac{(1+e^v)dv}{v+e^v} + \int\frac{dy}{y} = 0 \Rightarrow \ln(v+e^v)y = \ln c$$

$\Rightarrow (v + e^v)y = c \Rightarrow x + ye^{x/y} = c.$

7. (c) : $\cos 2\theta = \sin\theta \Rightarrow 1 - 2\sin^2\theta - \sin\theta = 0$

$\Rightarrow 2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

$\Rightarrow 2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -1, 1/2$

In $(0, 2\pi)$; $\sin\theta = -1$ for $\theta = \frac{3\pi}{2}$

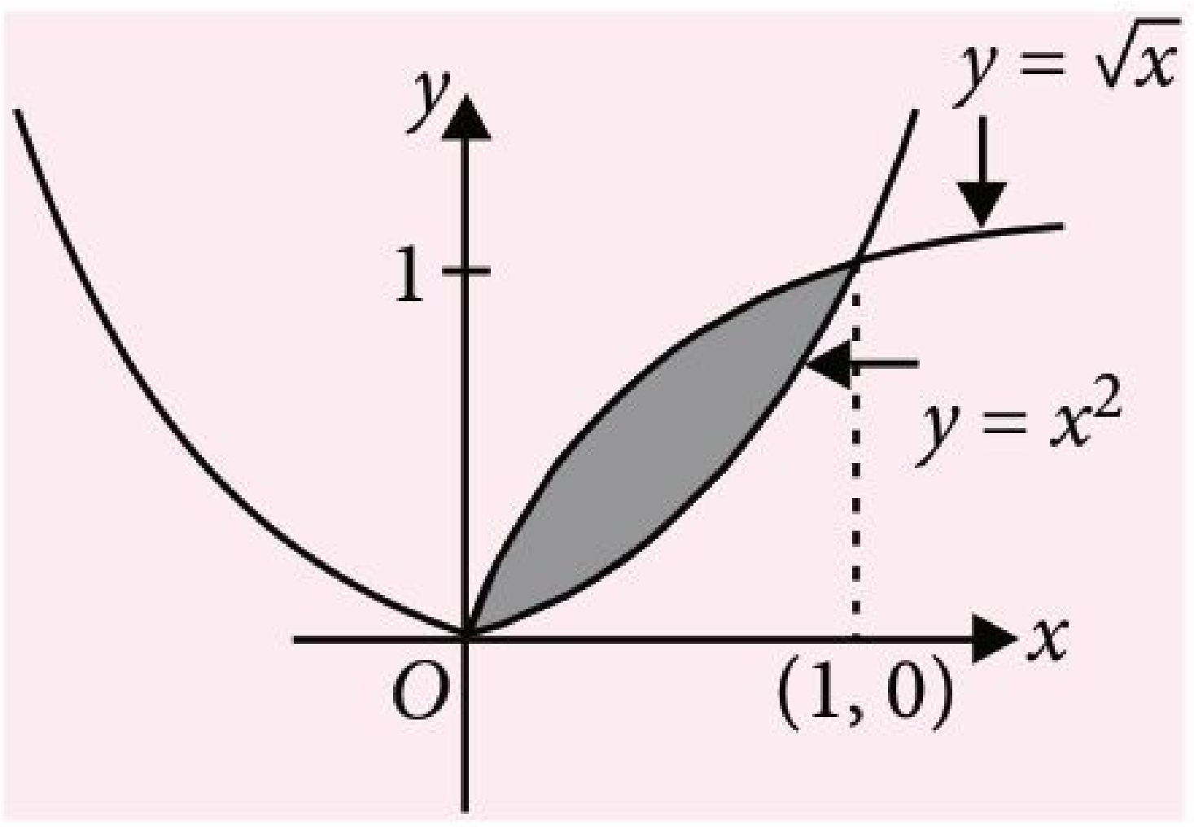
and $\sin\theta = \frac{1}{2}$ for $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ Total 3 solutions exist in $(0, 2\pi)$.

8. (c) : We have, $y = x^2$ ...(i) a parabola with vertex at (0, 0).

and $y = \sqrt{x}$ ...(ii) a half portion of parabola $y^2 = x$ having vertex at (0, 0).

Point of intersection of (i) and (ii) are (0, 0) and (1, 1)



$\therefore$ Required area = Area of shaded region

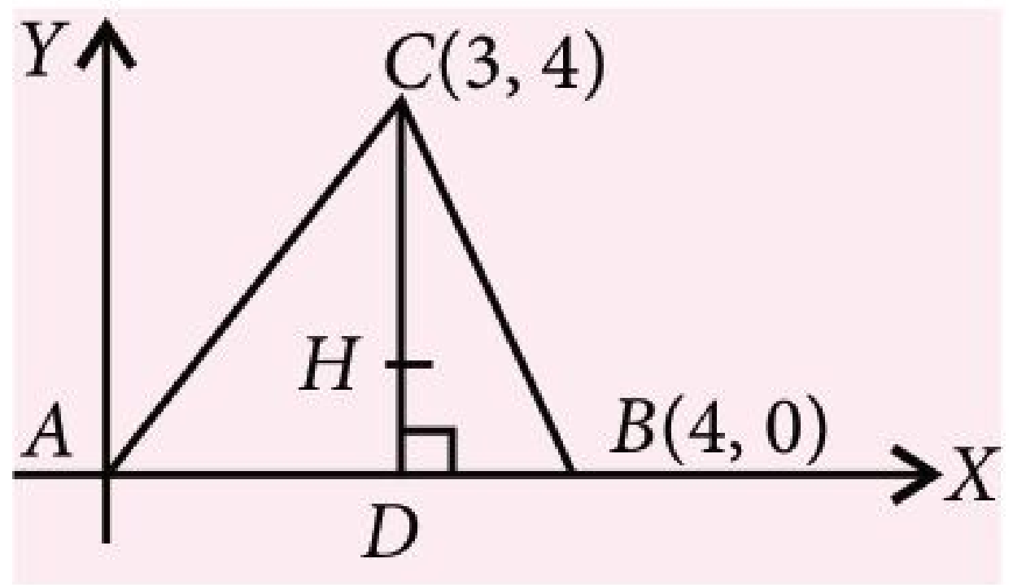
$$= \int_0^1 (\sqrt{x} - x^2)dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1 = \frac{1}{3} \text{ sq. unit}$$

9. (d) : $H(3, \alpha)$ is the orthocentre.

$\therefore \quad BH \perp AC$

$$\Rightarrow \left(\frac{0-\alpha}{4-3}\right)\left(\frac{4}{3}\right) = -1$$

$$\Rightarrow \alpha = \frac{3}{4}.$$



Thus, $H \equiv \left(3, \frac{3}{4}\right)$.

10. (c) : The ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$S_1 \equiv (-2, 0), S \equiv (2, 0)$

$\Rightarrow S_1S = 4 = 2ae = 2a \times \frac{1}{3} \Rightarrow a = 6$

$$b^2 = a^2(1-e^2) = 36\left(1-\frac{1}{9}\right) = 32$$

$$\Rightarrow \quad b = 4\sqrt{2},\ 2b = 8\sqrt{2}.$$

11. (a) : $\lim\limits_{x\to 0}\frac{2-\sqrt{x+4}}{\sin 2x} = \lim\limits_{x\to 0}\frac{(2-\sqrt{x+4})(2+\sqrt{x+4})}{\sin 2x(2+\sqrt{x+4})}$

$$= \lim_{x\to 0}\frac{4-x-4}{\sin 2x(2+\sqrt{x+4})}$$

$$= \lim_{x\to 0}\frac{-1}{2\left(\frac{\sin 2x}{2x}\right)(2+\sqrt{x+4})} = -\frac{1}{8}$$

12. (a) : 7 people can be seated in 6! ways.
Now, 2 persons can sit together in 2 × 5! ways.
So, required number of arrangements

$$= 6! - 2\times 5! = 720 - 240 = 480$$

13. (a) : We have, $e^{f(x)} = \frac{10+x}{10-x} \Rightarrow f(x) = \log\frac{10+x}{10-x}$

$$\therefore\ f\left(\frac{200x}{100+x^2}\right) = \log\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}$$

$$= \log\frac{x^2+20x+100}{x^2-20x+100} = \log\frac{(10+x)^2}{(10-x)^2} = 2\log\frac{10+x}{10-x} = 2f(x)$$

$$\therefore\ f(x) = \frac{1}{2}f\left(\frac{200x}{100+x^2}\right)$$

14. (b) : Let E_1, E_2, E_3 and A be the events defined as follows :

E_1 : the bag contains 2 white balls and 2 non-white balls,
E_2 : the bag contains 3 white balls and 1 non-white ball,
E_3 : the bag contains all four white balls and
A : two white balls have been drawn from the bag.

Then, $P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{3}$

$P(A|E_1)$ = probability of drawing 2 white balls when E_1 has occurred *i.e.* the bag contains 2 white and 2 non-white balls $= \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$

$P(A|E_2)$ = probability of drawing 2 white balls when E_2 has occurred *i.e.* the bag contains 3 white and 1 non-white ball $= \frac{{}^3C_2}{{}^4C_2} = \frac{3}{6} = \frac{1}{2}$

$P(A|E_3)$ = probability of drawing 2 white balls when E_3 has occurred *i.e.* the bag contains all four white balls

$$= \frac{{}^4C_2}{{}^4C_2} = 1$$

We want to find $P(E_3|A)$.
By Baye's theorem, we get

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{(1/3)\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1} = \frac{1}{\frac{1}{6}+\frac{1}{2}+1} = \frac{6}{10} = \frac{3}{5}$$

15. (c) : If $x \ge 0$ then $f(x) = x \ge 0$ and
if $x < 0$ then $f(x) = -x > 0$
$\therefore\ f(x) \in [0, \infty)\ \forall\ x \in R\ \therefore\ R(f) = [0, \infty)$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

6				9	8	3		7				4			1	5				6
			6		1									9	6		3			
			2					4							8					
	6		1		7	8								1	7		8	5	9	
7				4			6					9			5	1				3
1	8	5	9	3								5					9		6	7
	2		3							5	2						1			8
							4				7		6			8				
9				8	4				9		4							4		9
						6	3	7				5								
						4								9						
								8				7	4	2						
1		8							2		1				2	1				3
				3			1		7				2							
9			7						8	4							4		6	
5	6		4					9								2	5	7	3	4
8				9	3			6					8			7				1
	1	9	8		5	2								2	1		9		8	
					1							1					2			
			6		7	4									6		3			
2				4	9			7				3		8	4	5				2

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

16. (d) : We have, $2y = 3 - x^2 \Rightarrow 2\dfrac{dy}{dx} = -2x$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(1,1)} = -1$$

Equation of normal is, $(y-1) = \dfrac{1}{-(dy/dx)}(x-1)$

$\Rightarrow (y - 1) = 1(x - 1) \Rightarrow x - y = 0$

17. (d) : $\tan\left(\cos^{-1}\dfrac{4}{5} + \tan^{-1}\dfrac{2}{3}\right)$

$$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right) = \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}} = \frac{17}{6}.$$

18. (b) : Let d be the common difference

$\Rightarrow a_6 + a_9 + a_{12} + a_{15} = 20$

$\Rightarrow a_1 + 5d + a_1 + 8d + a_1 + 11d + a_1 + 14d = 20$

$\Rightarrow 4a_1 + 38d = 20 \Rightarrow 2a_1 + 19d = 10$

$$\Rightarrow S_{20} = \frac{20}{2}[2a_1 + 19d] = 10 \times 10 = 100.$$

19. (c) : $\vec{a} \times \vec{b} = 15\hat{i} - 10\hat{j} + 30\hat{k}$

and $|\vec{a} \times \vec{b}| = \sqrt{15^2 + (-10)^2 + (30)^2} = 35$

$\therefore$ Required vector $= \dfrac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

20. (d) : $\displaystyle\sum_{j=1}^{18}(x_j - 8) = 9 \Rightarrow \sum_{j=1}^{18} x_j = 153$

and $\displaystyle\sum_{j=1}^{18}(x_j - 8)^2 = 45$

$$\Rightarrow \sum_{j=1}^{18} x_j^2 = 45 - 64 \times 18 + 16 \times 153 = 1341$$

So, standard deviation $= \sqrt{\dfrac{\Sigma x_j^2}{n} - \left(\dfrac{\Sigma x_j}{n}\right)^2}$

$$= \sqrt{\frac{1341}{18} - \left(\frac{153}{18}\right)^2} = \sqrt{74.5 - 72.25} = 1.5$$

21. (8) : $A \equiv (t_1^2, 2t_1),\ B \equiv (t_2^2, 2t_2),$

$$t = -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$$

$$\Rightarrow t_1 - t_2 = \frac{2}{t_2} - \frac{2}{t_1} = \frac{2(t_1 - t_2)}{t_1 t_2} \Rightarrow t_1 t_2 = 2$$

$$t = -t_1 - \frac{2}{t_1} = -(t_1 + t_2)$$

$\Rightarrow t^2 = (t_1 + t_2)^2 \geq 4t_1t_2 = 4 \cdot 2 = 8$

22. (256) : Let $I = \displaystyle\int \sin 4x \cdot e^{\tan^2 x} dx$

$$= \int 2\sin 2x \cos 2x \cdot e^{\tan^2 x} dx$$

$$= 4\int \sin x \cos x \cdot \left(\frac{1 - \tan^2 x}{1 + \tan^2 x}\right) \cdot e^{\tan^2 x} dx$$

$$= 4\int \frac{\tan x \sec^2 x (1 - \tan^2 x) \cdot e^{\tan^2 x}}{\sec^6 x} dx$$

Putting $\tan^2 x = t \Rightarrow 2\tan x \sec^2 x\, dx = dt$, we get

$$I = 2\int \frac{(1-t)e^t dt}{(1+t)^3} = -2\int \frac{\{(t+1)-2\}e^t dt}{(t+1)^3}$$

$$= -2\int e^t\left(\frac{1}{(1+t)^2} - \frac{2}{(1+t)^3}\right)dt = -\frac{2e^t}{(1+t)^2} + c$$

$$= \frac{-2e^{\tan^2 x}}{\sec^4 x} + c = -2\cos^4 x\, e^{\tan^2 x} + c$$

$\therefore\ a = -2, b = 4$, then $a^{2b} = (-2)^8 = 256$

23. (2.8) : $12^{4+2x^2} = (24\sqrt{3})^{3x^2-2} \Rightarrow (12)^{4+2x^2} = (12)^{\frac{3}{2}(3x^2-2)}$

$\therefore\ 4 + 2x^2 = \dfrac{3}{2}(3x^2 - 2) \Rightarrow 5x^2 = 14 \Rightarrow x^2 = \dfrac{14}{5} = 2.8$

24. (3) : Let $\vec{v}_1 = \hat{i} - 2\hat{j} + \hat{k},\ \vec{v}_2 = 3\hat{i} + 2\hat{j} - \hat{k}$

and $\vec{v}_3 = \vec{c} = \alpha\vec{a} + \beta\vec{b} = \alpha(\hat{i} + \hat{j}) + \beta(\hat{j} + \hat{k})$

$= \alpha\hat{i} + (\alpha + \beta)\hat{j} + \beta\hat{k}$

Since, $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are coplanar. [Given]

$$\Rightarrow \begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ \alpha & \alpha+\beta & \beta \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - (C_1 + C_3)$, we have

$$\begin{vmatrix} 1 & -4 & 1 \\ 3 & 0 & -1 \\ \alpha & 0 & \beta \end{vmatrix} = 0 \Rightarrow 4(3\beta + \alpha) = 0 \Rightarrow \frac{\alpha}{\beta} = -3 \therefore \left|\frac{\alpha}{\beta}\right| = 3$$

25. (6) : Let a_n be the number of divisions made by n lines.

By drawing the lines, we find

$a_0 = 1, a_1 = 2, a_2 = 4, a_3 = 7, a_4 = 11$,.....

$a_1 - a_0 = 1; a_2 - a_1 = 2; a_3 - a_2 = 3; a_4 - a_3 = 4$; ....,

$a_{10} - a_9 = 10$

Adding, $a_{10} - a_0 = 1 + 2 + 3 + \ldots + 10 = 55$

$\therefore\ a_{10} = a_0 + 55 = 1 + 55 = 56$

26. (2) : Let a, b, c be the DR's of line

$3x - 2y + z + 3 = 0 = 4x - 3y + 4z + 1$

So, $3a - 2b + c = 0$ and $4a - 3b + 4c = 0$

$\Rightarrow \quad a : b : c = 5 : 8 : 1$

$\because \quad$ Dr's of normal to the plane $2x - y + mz - 2 = 0$ are $2, -1, m$.

For line to be parallel to plane, line must be perpendicular to the normal to the plane.

$5 \cdot 2 + 8 \cdot (-1) + 1 \cdot m = 0 \quad \Rightarrow \quad m = -2$

27. (4) : Given, $f(x) = x^2 + ax + 5 \Rightarrow f'(x) = 2x + a$

Since $x \in [1, 2] \Rightarrow 1 \le x \le 2$

$\Rightarrow \quad 2 \le 2x \le 4 \Rightarrow 2 + a \le 2x + a \le 4 + a$

Since, $f(x)$ is increasing in $[1, 2]$

$\Rightarrow \quad f'(x) \ge 0 \quad \Rightarrow \quad 2 + a \ge 0 \Rightarrow a \ge -2$

$\therefore \quad$ Minimum value of a is $-2 = \dfrac{-k}{2} \Rightarrow k = 4$

28. (3) : $3^{100} = 9^{50} = (10 - 1)^{50} = (1 - 10)^{50}$

$$= 1 - 50 \cdot 10 + \frac{50 \cdot 49}{2} \cdot 10^2 - \frac{50 \cdot 49 \cdot 48}{6} \cdot 10^3 + \text{multiple of } 10^4.$$

$= 1 - 500 + 122500 +$ multiple of 10^4

$= 122001 +$ multiple of 10^4.

The sum of last 4 digits $= 2 + 0 + 0 + 1 = 3$.

29. (0.5) : Since, $P(Y \mid X) = \dfrac{P(X \cap Y)}{P(X)}$, so we have

$$P(X \cap Y) = P(Y \mid X) \cdot P(X) = \frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}$$

Now, $P(X \mid Y) = \dfrac{P(X \cap Y)}{P(Y)} = \dfrac{2/15}{4/15} = \dfrac{1}{2}$

$\therefore \quad P(X' \mid Y) = (1 - P(X \mid Y)) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

30. (5) : We have, $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$

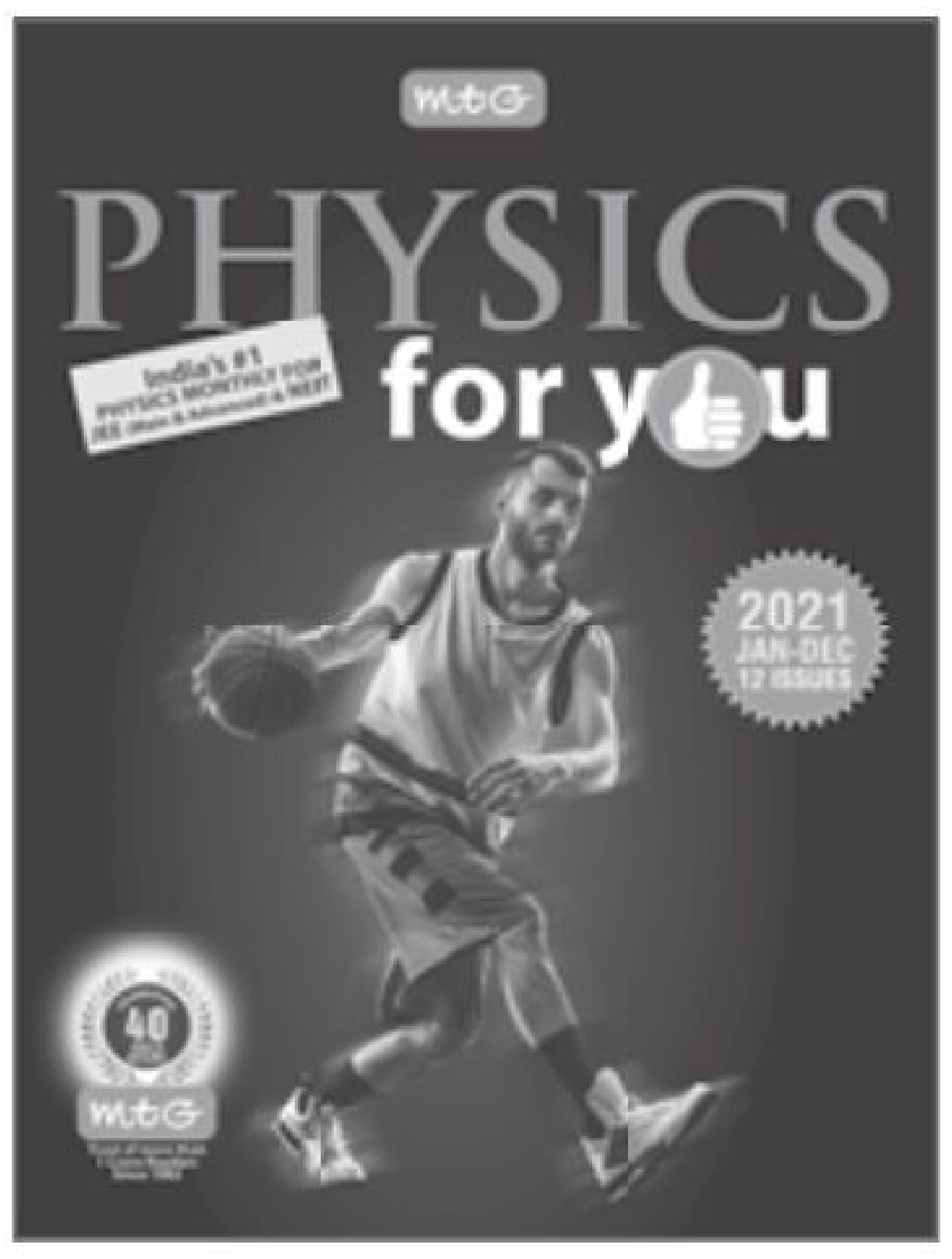
Now, $A^2 - kA - 5I = O$

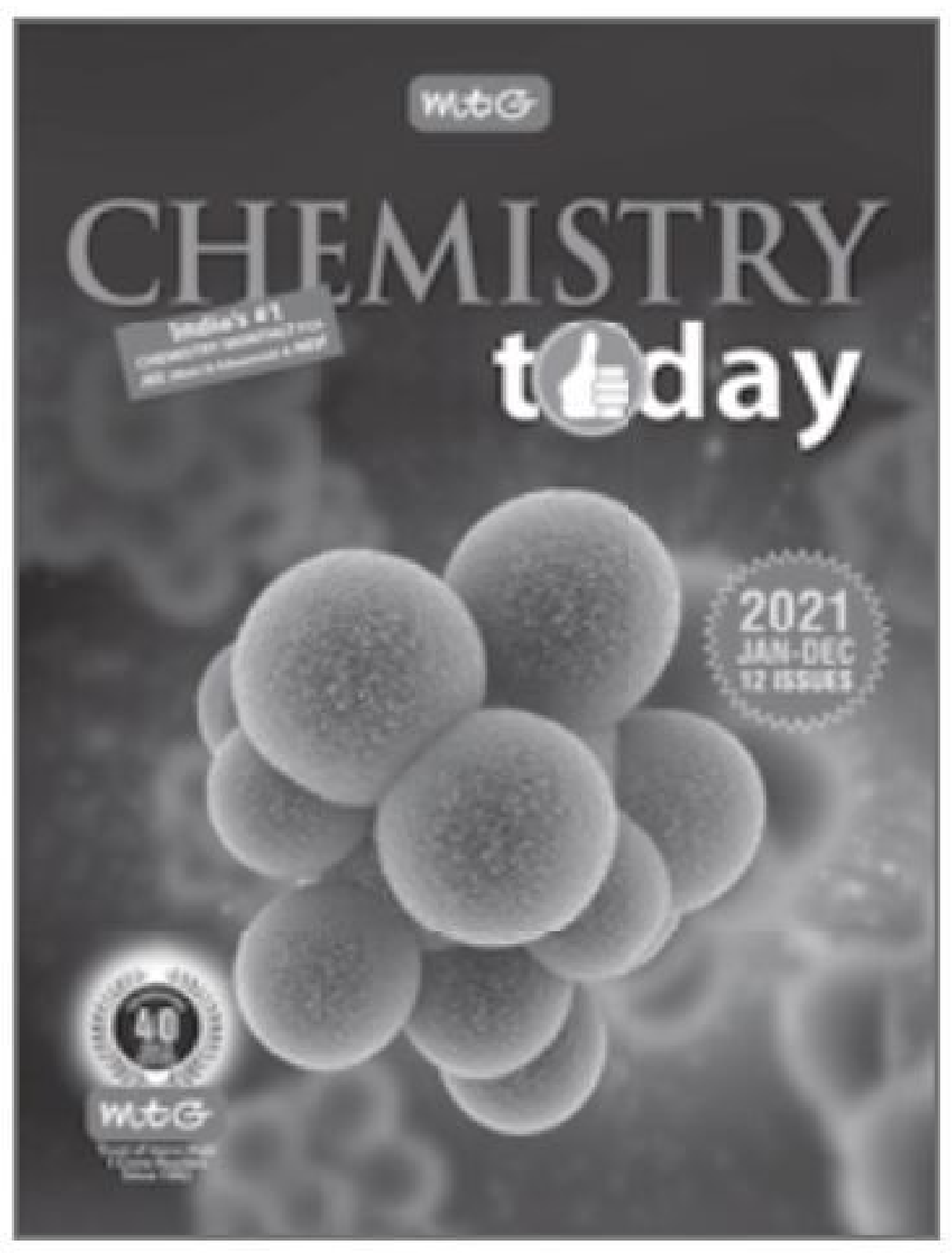
$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} k & 3k \\ 3k & 4k \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

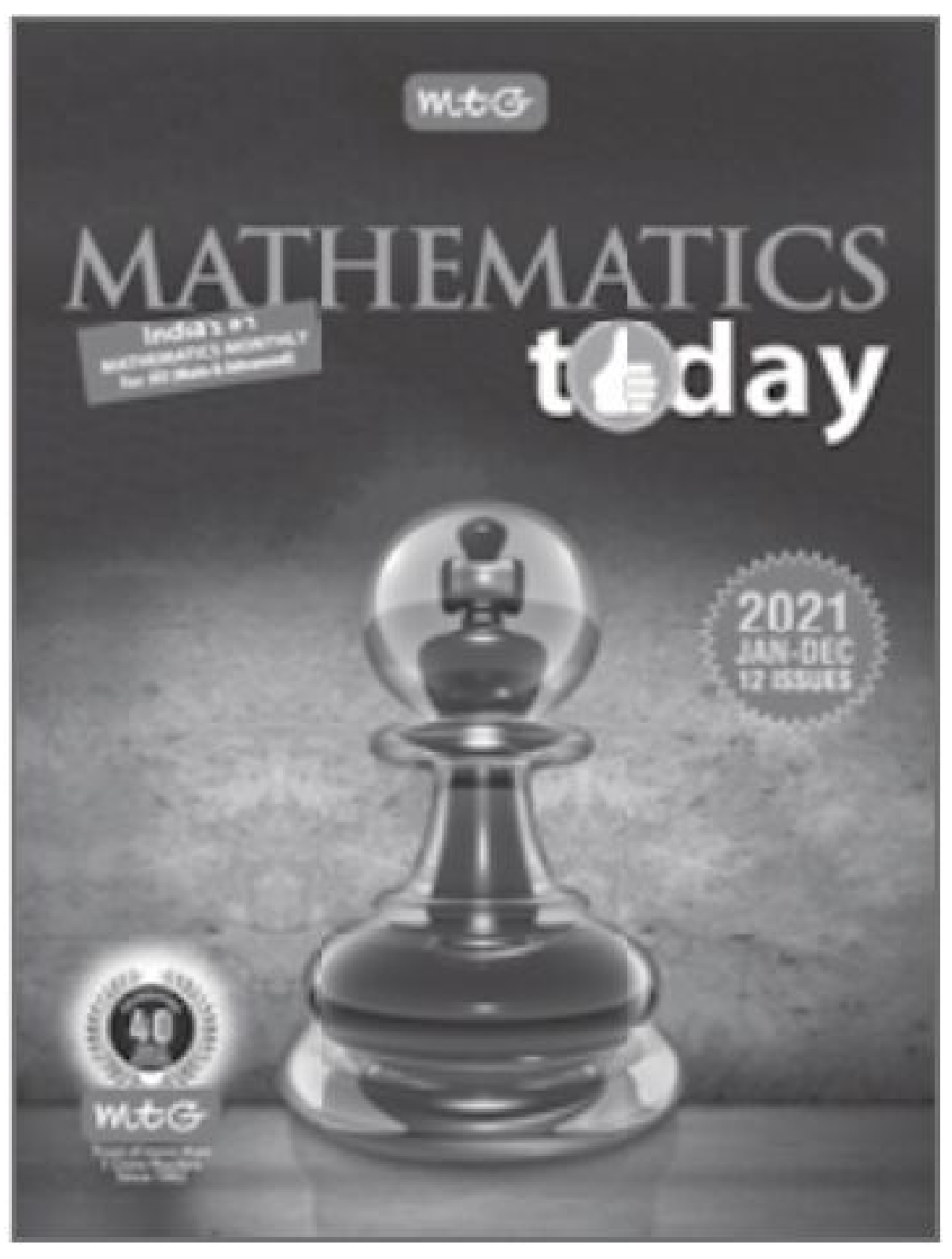
$$\Rightarrow \begin{bmatrix} 5-k & 15-3k \\ 15-3k & 20-4k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

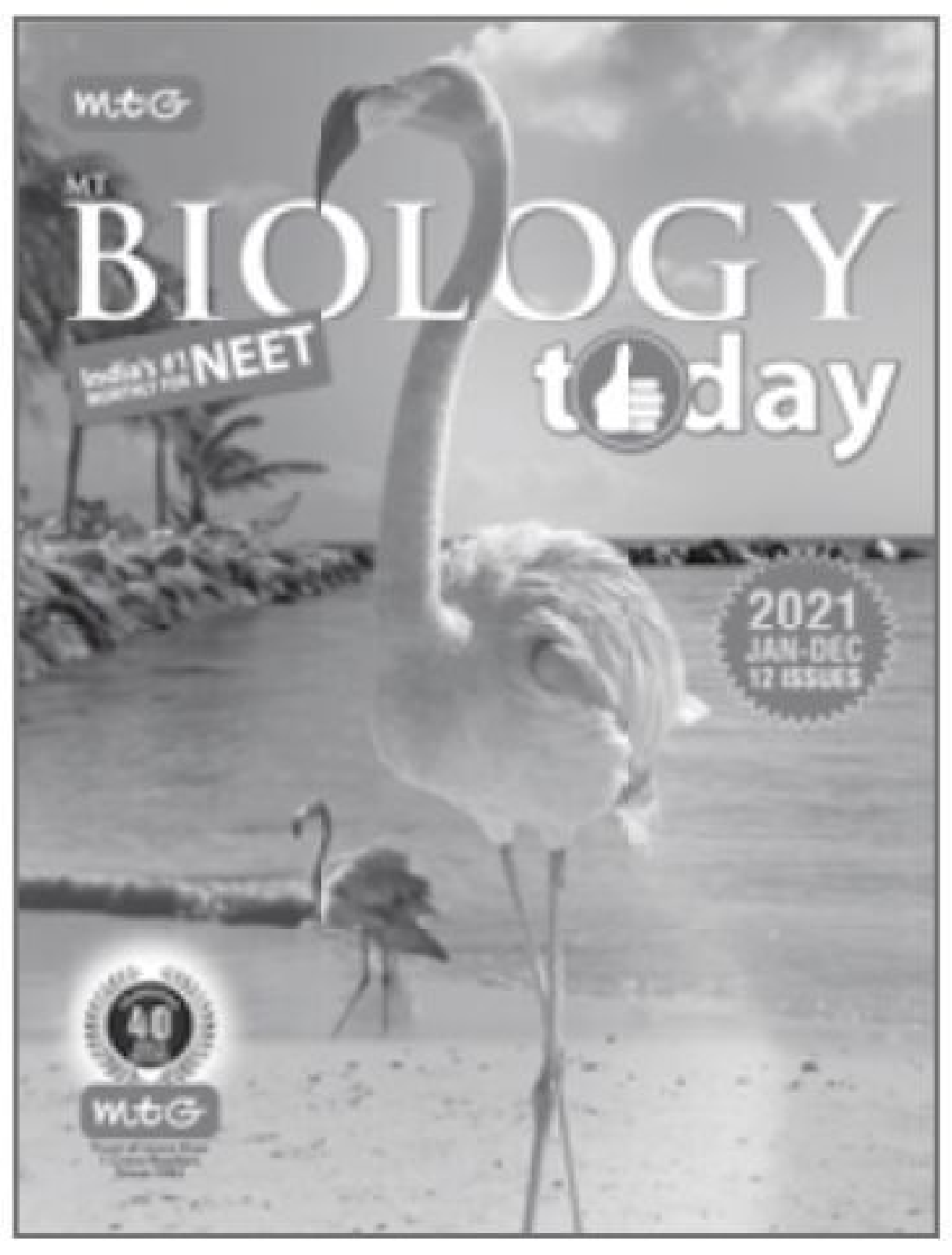
$\Rightarrow \quad 5 - k = 0 \,;\, 15 - 3k = 0$ and $20 - 4k = 0 \;\Rightarrow\; k = 5$

◆ ◆









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JEE 2022 PRACTICE PAPER ADVANCED

Exam on 28th August

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. $f:\left(0,\frac{\pi}{2}\right)\to A$, $f(x)=\log_e(\sin x^{\sin x}+1)$, then the minimum value of $f(x)$ is

(a) $\log_e 2$ (b) $\log_e\left(\left(\frac{1}{e}\right)^{1/e}+1\right)$

(c) $\log_e((e)^2+1)$ (d) 2

2. If $x^5-x^3+x=a$, then minimum value of x^6 is

(a) a (b) $2a-1$

(c) $a+2$ (d) $a-3$

3. The value of $\lim\limits_{n\to\infty}\left(\frac{n!}{(mn)^n}\right)^{1/n}$ is

(a) em (b) $\frac{e}{m}$

(c) $\frac{1}{em}$ (d) none of these

4. The set of values of a for which inequation $(a-1)x^2-(a+1)x+a-1\geq 0$ is true for all $x\geq 2$ is

(a) $\left(1,\frac{7}{3}\right]$ (b) $(-\infty,1)$

(c) $\left[\frac{7}{3},\infty\right)$ (d) none of these

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

There are 10 players of a game, 6 men and 4 women.

5. The number of singles matches that can be arranged among them is

6. The number of doubles matches that can be arranged among them is

Question Stem for Question Nos. 7 and 8

If $\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix}=0$, $x\neq a$, $y\neq b$, $z\neq c$ then

7. The value of $\frac{x+a}{x-a}+\frac{y+b}{y-b}+\frac{z+c}{z-c}$ is equal to

8. The value of $\frac{x}{x-a}+\frac{y}{y-b}+\frac{z}{z-c}$ is equal to

Question Stem for Question Nos. 9 and 10

$[\vec{a}\,\vec{b}\,\vec{c}]=\vec{a}\cdot(\vec{b}\times\vec{c})$ and $\vec{a}\perp\vec{b}\Rightarrow\vec{a}\cdot\vec{b}=0$.

9. If $\vec{u}=(q-r,r-p,p-q)$ and $\vec{v}=\left(\frac{1}{a},\frac{1}{b},\frac{1}{c}\right)$ and a, b, c are T_p, T_q, T_r of an H.P. then the angle between the vectors $\vec{u}$ and $\vec{v}$ (in degrees) is

10. If $\vec{u}=(q-r,r-p,p-q)$ and $\vec{v}=(\log a^2,\log b^2,\log c^2)$ and a,b,c are T_p, T_q, T_r of a G.P., then angle between vectors $\vec{u}$ and $\vec{v}$ (in degree) is

SECTION 3

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*

11. If the system of equations $x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solutions, then the possible values of k are

(a) $-1, 2$ (b) $0, 1$ (c) $1, 2$ (d) $-1, 1$

12. If both the roots of equation $ax^2+x+c-a=0$ are imaginary and $c>-1$, then

(a) $3a>2+4c$ (b) $a>0$

(c) $c<a$ (d) None of these

13. The solution of the equation

$$3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28$$

(a) $(2n+1)\frac{\pi}{2}, n \in I$ (b) $n\pi - \frac{\pi}{4}, n \in I$

(c) $2n\pi$ (d) none of these

14. If $\sin y = x\cos(a+y)$, then $\frac{dy}{dx}$ is

(a) $\frac{\cos^2(a+y)}{\cos a}$ (b) $\frac{\cos(a+y)}{x\sin(a+y)+\cos y}$

(c) $\frac{\cos a}{3x^2-2x\cos a-1}$ (d) none of these

15. The range of the function $y=\sqrt{2\{x\}-\{x\}^2-\frac{3}{4}}$ is (where {.} denotes fractional part)

(a) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (b) $\left[0, \frac{1}{2}\right)$

(c) $\left[0, \frac{1}{4}\right]$ (d) $\left[\frac{1}{4}, \frac{1}{2}\right]$

16. If the equation $\bar{a}z + a\bar{z} + b = 0$ and $\bar{a}z - a\bar{z} + b_1 = 0$ represent two lines L_1 and L_2 in the complex plane then

(a) L_1 and L_2 are perpendicular

(b) b is purely real

(c) b_1 is purely imaginary

(d) b_1 is purely real

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. In ΔABC if BC is unity,

$\sin\frac{A}{2}=x_1, \sin\frac{B}{2}=x_2, \cos\frac{A}{2}=x_3$ and $\cos\frac{B}{2}=x_4$

with $\left(\frac{x_1}{x_2}\right)^{2007} - \left(\frac{x_3}{x_4}\right)^{2006} = 0$, then the length of AC is

18. If $I_n = \int_0^\infty e^{-x}(\sin x)^n\,dx\ (n>1)$, then the value of $\frac{101 I_{10}}{90 I_8}$ is equal to

19. The A.M., G.M. and H.M. of first and last term of the series 100, 101, 102,...., n are also the term of this series, then the value of $\frac{n}{200}$ $(100 < n \leq 500)$ is

PAPER-2

SECTION 1

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*

1. If m_1 and m_2 be the slopes of two perpendicular chords of equal length passing through origin of circle $(x-1)^2+(y+2)^2=5$, then the value of $m_1^2+m_2^2$ is equal to

(a) 80/9 (b) 82/9

(c) 83/9 (d) none of these

2. If $[x]$ represents the greatest integer less than or equal to x, then which of the following statement is true?

(a) $\sin[x] = \cos[x]$ has no solution

(b) $\sin[x] = \tan[x]$ has infinitely many solutions

(c) $\sin[x] = \cos[x]$ possess unique solution

(d) $\sin[x] = \tan[x]$ for no value of x

3. The $\det \Delta = \begin{vmatrix} d^2+r & de & df \\ de & e^2+r & ef \\ df & ef & f^2+r \end{vmatrix}$ is divisible by

(a) r (b) $(d+e^2+f^2+r)$

(c) $(d^2+e^2+f^2+r)$ (d) $(d^2+e+f^2+r^2)$

4. The largest term of $a_n = \frac{n^2}{n^3+200}$, $n \in N$ is

(a) $\frac{29}{453}$ (b) $\frac{49}{543}$ (c) $\frac{43}{553}$ (d) $\frac{41}{451}$

5. If $L = \lim_{x\to 0}\frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}}-\frac{1+ax}{1+bx}\right)$ exists, then

(a) $a=\frac{1}{4}$ (b) $b=\frac{3}{4}$

(c) $L=-\frac{1}{32}$ (d) $L=\frac{1}{32}$

6. If $X \cup \{1, 2\} = \{1, 2\ 3, 5, 9\}$, then

(a) the smallest set X is $\{3, 5, 9\}$

(b) the smallest set X is $\{2, 3, 5, 9\}$

(c) the largest set X is $\{1, 2, 3, 5, 9\}$

(d) the largest set X is $\{2, 3, 4, 9\}$

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 7 and 8

Consider the sets $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

7. The probability of an increasing functions from A to B is

8. The probability of non decreasing functions from A to B is

Question Stem for Question Nos. 9 and 10

Let P be a variable point on the ellipse $x^2 + 2y^2 = 2$ and S, S_1 be the foci.

9. The locus of the centroid of triangle PSS_1 is a conic of eccentricity $e =$

10. The locus of the incentre I of triangle PSS_1 is a conic of eccentricity $e = \sqrt{n\sqrt{n} - n}$ then n is ________.

Question Stem for Question Nos. 11 and 12

Consider the function $f(x)$ such that

$$f(x) = x^2 + \int_0^1 (x+t) f(t)\, dt$$

11. The area bounded by the x-axis, the tangent and normal at $x = 0$ on the curve $y = f(x)$ is

12. The area bounded by the curve $y = f(x)$ and the x-axis is $\frac{n}{18}\sqrt{\frac{n}{3}}$ then n is ________.

SECTION 3

- ***This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***

Paragraph for Q. 13 and Q. 14

A ray of light is coming along the line $L = 0$ and strikes the plane mirror kept along the plane $P = 0$ at B. $A(2, 1, 6)$ is a point on the line $L = 0$ whose image about $P = 0$ is A'. It is given that $L = 0$ is $\frac{x-2}{3} = \frac{y-1}{4} = \frac{z-6}{5}$ and $P = 0$ is $x + y - 2z = 3$.

13. Co-ordinates of A' are

(a) $(6, 5, 2)$ (b) $(6, 5, -2)$
(c) $(6, -5, -2)$ (d) none of these

14. Co-ordinates of B are

(a) $(5, 10, 6)$ (b) $(10, 15, 11)$
(c) $(-10, -15, -14)$ (d) none of these

Paragraph for Q. 15 and Q. 16

Consider an ellipse $\frac{x^2}{4} + y^2 = \alpha$, ($\alpha$ is parameter > 0) and a parabola $y^2 = 8x$. A common tangent to the ellipse and the parabola meets the co-ordinate axes at A & B.

15. Locus of mid point of AB is

(a) $y^2 = -2x$ (b) $y^2 = -x$
(c) $y^2 = -\frac{x}{2}$ (d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

16. If the eccentric angle of a point on the ellipse where the common tangent meets it is $\left(\frac{2\pi}{3}\right)$, then α is equal to

(a) 4 (b) 5 (c) 26 (d) 36

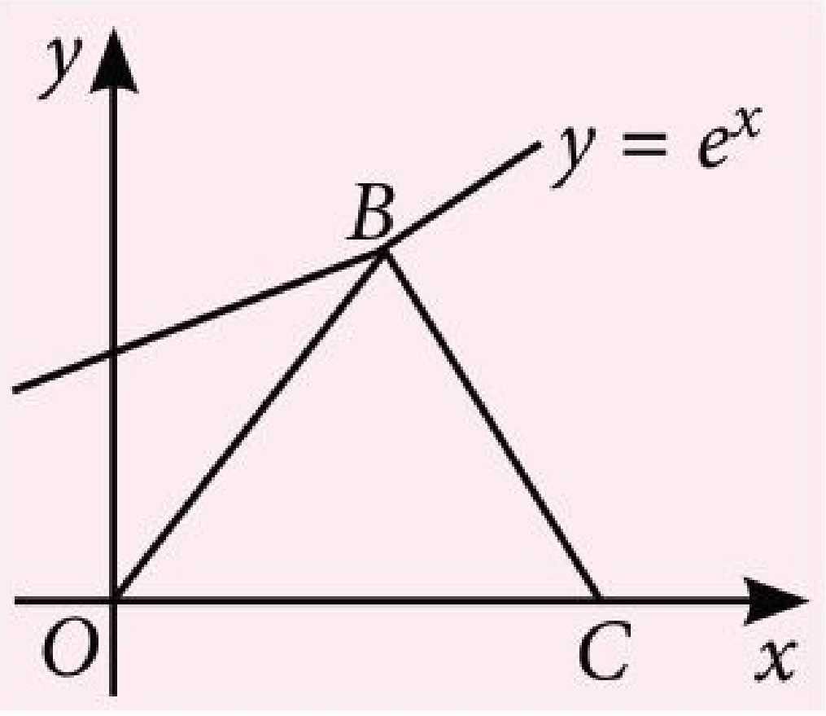
SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. If $f : [2, \infty) \to R$; $f(x)$ is function which is differentiable as well as increasing and $f(2) = 2$; then the number of solution of equation $f(f(x)) = \frac{4}{(x^2 - 4x + 6)}$ is:

18. Let point B is a variable point (α, e^{α}) lying on curve $y = e^x$, where $0 \le \alpha \le \ln 2$. If ΔOBC as shown in figure is such that $OB = OC$ and C lies on x-axis.

If maximum area of ΔOBC is $\lambda\sqrt{(\ln 2)^2 + 4}$, then $\left[\frac{100\lambda}{99}\right]$ (where [.] denotes greatest integer function)



19. If absolute maximum value of $f(x) = \frac{1}{|x| + 5} + \frac{1}{|x - 5| + 5}$ is A, then $10A$ is:

EXAM ALERT 2022

Exam	Date
JEE Main	Session 1: 20th to 29th June Session 2: 21st to 30th July
COMEDK	19th June
SRMJEEE	Phase 3 : 25th and 26th June
BITSAT	Session-I: 2nd to 9th July Session-II: 3rd to 7th August
KEAM	4th July
VITEEE	30th June to 6th July
JEE Advanced	28th August

SOLUTIONS

PAPER-1

1. (b) : Let $h(x) = \log(x^x + 1)$, for $0 < x < 1$, $h'(x) > 0$ if $x > \frac{1}{e}$. If $x > \frac{1}{e}$, for $0 < x < 1$, function will be increasing, then it will be one-one for $x \in (0, 1)$. $h'(x) < 0$, if $x < \frac{1}{e}$, so, at $x = \frac{1}{e}$, there is minimum.

$$f_{\min} = \log_e\left(\left(\frac{1}{e}\right)^{1/e} + 1\right).$$

2. (b) : $x(x^4 - x^2 + 1) = \frac{x(x^6+1)}{x^2+1}$.

Suppose $x > 0$, $a > 0$,

$$x^6 + 1 = \frac{a(x^2+1)}{x} = a\left(x + \frac{1}{x}\right) \ge 2a, x^6 \ge 2a - 1.$$

If $x \le 0 \Rightarrow a \le 0$, $2a - 1 < 0 \le x^6 \Rightarrow x^6 \ge 2a - 1$.

3. (c) : Let $L = \lim\limits_{n\to\infty} \frac{1}{m}\left(\frac{1}{n}.\frac{2}{n}.\frac{3}{n}...\frac{n}{n}\right)^{1/n}$

$$\Rightarrow \ln L = \lim_{n\to\infty}\left[\ln\left(\frac{1}{m}\right) + \frac{1}{n}\left(\ln\frac{1}{n} + \ln\frac{2}{n} + \ldots\ldots + \ln\frac{n}{n}\right)\right]$$

$$= -\ln m + \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right)$$

$$= -\ln m + \int_0^1 \ln x\,dx = -\ln m - 1 = \ln\left(\frac{1}{em}\right) \therefore L = \frac{1}{em}.$$

4. (c) : Given inequation is $(a - 1)x^2 - (a + 1)x + a - 1 \ge 0$

$$\Rightarrow a(x^2 - x + 1) - (x^2 + x + 1) \ge 0$$

$$\Rightarrow a \ge \frac{x^2+x+1}{x^2-x+1} = 1 + \frac{2x}{x^2-x+1} = 1 + \frac{2}{x + \frac{1}{x} - 1} \quad \ldots\text{(i)}$$

Let $y = x + \frac{1}{x}$, y is increasing in $[2, \infty)$

$\Rightarrow 1 + \frac{2}{x + \frac{1}{x} - 1} \in \left(1, \frac{7}{3}\right]$ for all $x \ge 2$ equation (i) should be true.

$$\Rightarrow a \ge \frac{7}{3}.$$

5. (45) : The desired number is $\binom{10}{2} = 45$.

6. (630) : By choosing any four a, b, c, d we can have 3 matches, $ab - cd$, $ac - bd$, $ad - bc$.

The desired number is $\binom{10}{4} \cdot 3 = 210 \times 3 = 630$

(7 – 8) :

7. (–1) **8. (1)**

$$\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ gives

$$\Rightarrow \begin{vmatrix} a & y & z \\ x-a & b-y & 0 \\ x-a & 0 & c-z \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \frac{a}{x-a} & \frac{y}{y-b} & \frac{z}{z-c} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{a}{x-a} - \frac{y}{y-b}(-1) + \frac{z}{z-c}(1) = 0$$

$$\Rightarrow \left(\frac{x}{x-a} - 1\right) + \frac{y}{y-b} + \frac{z}{z-c} = 0$$

$$\Rightarrow \frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c} = 1 \Rightarrow \frac{2x}{x-a} + \frac{2y}{y-b} + \frac{2z}{z-c} = 2$$

$$\Rightarrow \frac{x+a}{x-a} + 1 + \frac{y+b}{y-b} + 1 + \frac{z+c}{z-c} + 1 = 2$$

$$\Rightarrow \frac{x+a}{x-a} + \frac{y+b}{y-b} + \frac{z+c}{z-c} = -1$$

9. (90) : $1/a = T_p$ of A.P. $= A + (p - 1)D = (A - D) + pD$.
If θ be the angle between $\vec{u}$ and $\vec{v}$, then

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}| \cos\theta = \Sigma(q - r)\,[(A - D) + pD]$

$= (A - D)\,\Sigma\,(q - r) + D\Sigma\, p\,(q - r) = 0$

$\therefore \cos\theta = 0$ or $\theta = \pi/2$

10. (90) : $a = AR^{p-1}$

$\therefore \log a = \log A + (p - 1)\log R$

If θ be the angle between $\vec{u}$ and $\vec{v}$, then

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}| \cos\theta = \Sigma(q - r)\log a^2 = 2\Sigma(q - r)\log a$

$= 2\Sigma(q - r)\,[\log A + (p - 1)\log R]$

$= 2[\log A\Sigma(q - r) + \log R\Sigma(p - 1)(q - r)] = 0$

$\therefore \cos\theta = 0$ or $\theta = \pi/2$

11. (d) : Set of homogeneous equations will have non-trivial solution if $\Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -k & 0 \\ k & -1 & k-1 \\ 1 & 1 & 0 \end{vmatrix} = 0 \qquad [\text{Applying } C_3 \to C_3 + C_1]$$

$\Rightarrow \ k^2 - 1 = 0 \ \Rightarrow \ k = \pm 1$

12. (b) : Here $f(1) = a + 1 + c - a$.
As $c > -1 \ \Rightarrow \ f(1) > 0 \Rightarrow a > 0$

13. (a, b) : We have,

$$3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28, \ y + \frac{27}{y} = 28,$$

$\Rightarrow \ y^2 - 28y + 27 = 0 \Rightarrow y = 1$

$(\because \ y \neq 27$, since $\sin 2x + \cos 2x \leq \sqrt{2}$)

$\therefore \ \sin 2x + 2\cos^2 x = 0 \Rightarrow 2\cos x(\sin x + \cos x) = 0$

$$\Rightarrow \ x = (2n+1)\frac{\pi}{2} \text{ or } x = n\pi - \frac{\pi}{4}$$

14. (a, b) : $\sin y = x\cos(a+y) \Rightarrow \dfrac{\sin y}{\cos(a+y)} = x$ ...(i)

$$\frac{dx}{dy} = \frac{\cos y \cos(a+y) + \sin y \sin(a+y)}{\cos^2(a+y)} = \frac{\cos a}{\cos^2(a+y)}$$

$$\Rightarrow \ \frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a} \qquad \text{...(ii)}$$

$$= \frac{\cos^2(a+y)}{\cos y \cos(a+y) + \sin y \sin(a+y)} = \frac{\cos(a+y)}{\cos y + x\sin(a+y)}$$

[Using (i)]

15. (b) : Set $\{x\} = t$ so that $0 \leq t < 1$, $y = \sqrt{-t^2 + 2t - \dfrac{3}{4}}$

The domain of definition is $\dfrac{1}{2} \leq t < 1$

$$y\left(\frac{1}{2}\right) = \sqrt{-\frac{1}{4} + 1 - \frac{3}{4}} = 0; \ y(1) = \sqrt{-1 + 2 - \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

But $\dfrac{1}{2}$ is not attained, so the range is $\left[0, \dfrac{1}{2}\right)$.

16. (a, b, d)

17. (1) : In given ΔABC both $\dfrac{A}{2}$ and $\dfrac{B}{2}$ lie strictly between $\left(0, \dfrac{\pi}{2}\right)$ and $\sin x$ is always increasing in $\left(0, \dfrac{\pi}{2}\right)$ whereas $\cos x$ is always decreasing throughout $\left(0, \dfrac{\pi}{2}\right)$.

So if $\dfrac{A}{2} > \dfrac{B}{2} \Rightarrow \sin\dfrac{A}{2} > \sin\dfrac{B}{2} \Rightarrow x_1 > x_2$

and $\dfrac{1}{x_3} > \dfrac{1}{x_4}$. Now, $x_1^{2007} x_4^{2006} = x_2^{2007} x_3^{2006}$ is not valid.

Similarly for $\dfrac{A}{2} < \dfrac{B}{2}$

$\sin\dfrac{A}{2} < \sin\dfrac{B}{2} \Rightarrow x_1 < x_2$ and $\dfrac{1}{x_3} < \dfrac{1}{x_4}$. Again equality is not possible.

Therefore only possible case is when $\dfrac{A}{2} = \dfrac{B}{2}$

$$\Rightarrow \ x_1 = x_2 \text{ and } \frac{1}{x_3} = \frac{1}{x_4}.$$

Hence in this case ΔABC is isosceles with $\angle ABC = \angle CAB$

18. (1) : $l_n = \left(\dfrac{e^{-x}(\sin x)^n}{-1}\right)_0^\infty + \displaystyle\int_0^\infty n(\sin x)^{n-1}\cos x e^{-x} dx$

$$= 0 + n\left(\frac{(\sin x)^{n-1}\cos x e^{-x}}{-1}\right)_0^\infty$$

$$+ n\int_0^\infty \left[(\sin x)^{n-1}(-\sin x) + (\cos x)(n-1)\cos x(\sin x)^{n-2}\right]e^{-x} dx$$

$$= n\int_0^\infty \left[-(\sin x)^n + (n-1)(1 - \sin^2 x)(\sin x)^{n-2}\right]e^{-x} dx$$

$$= \frac{n(n-1)}{n^2+1} l_{n-2}, \ \frac{101}{90}\frac{l_{10}}{l_8} = \frac{101}{90} \times \frac{10 \times 9}{10^2+1} = 1$$

19. (2) : $\text{A.M.} = \dfrac{100+n}{2}$, $\text{G.M.} = \sqrt{100 \times n}$,

$$\text{H.M.} = \frac{200n}{100+n}$$

Let $n = 4k^2$ as for A.M and G.M we need a even perfect square

$$\therefore \ \text{H.M.} = \frac{800k^2}{100 + 4k^2} = \frac{200k^2}{25 + k^2}$$

Now, $100 < \dfrac{200k^2}{25+k^2} \leq 500 \ \Rightarrow \ 2500 + 100k^2 \leq 200k^2$

$\Rightarrow \ 2500 \leq 100\,k^2$ and $200k^2 \leq 12500 + 500k^2$

$\Rightarrow \ -300k^2 \leq 12500 \Rightarrow k^2 = 25, 36, 49, 64, 81, 100$,

H.M. is an integer for $k^2 = 100$ only

$\Rightarrow \ n = 4k^2 = 4 \times 100 = 400$.

PAPER-2

1. (b) : Let slope of chord, $OA = m$, $\tan 45° = \left|\dfrac{m+2}{1-2m}\right|$

$$\Rightarrow \ |m+2| = |1-2m| \Rightarrow m+2 = \pm(1-2m) \Rightarrow m = 3 \text{ or } -\frac{1}{3}$$

$$\Rightarrow \ m_1^2 + m_2^2 = (m_1+m_2)^2 - 2m_1m_2 = \frac{64}{9} + 2 = \frac{82}{9}$$

2. (a, b) : Since, according to option (a), we have $\sin[x] = \cos[x]$

$$\Rightarrow \quad [x] = n\pi + \left(\frac{\pi}{4}\right) = \left(n + \frac{1}{4}\right)\pi$$

Which is not possible as LHS $\in Z$ and RHS is irrational. Hence, $\sin[x] = \cos[x]$ has no solution.

However, $\sin[x] = \tan[x] \Rightarrow [x] = n\pi,\ n \in I$ which is possible when $[x] = 0$ and $n = 0$ $\therefore$ $0 \le x < 1$.

3 **(a, c) :** Multiply R_1, R_2, R_3 by d, e, f and take out d, e, f common from C_1, C_2, C_3, then $\Delta = \begin{vmatrix} d^2+r & d^2 & d^2 \\ e^2 & e^2+r & e^2 \\ f^2 & f^2 & f^2+r \end{vmatrix}$

Operate $R_1 \to R_1 + R_2 + R_3$ and take out $d^2 + e^2 + f^2 + r$ common from R_1, then

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ e^2 & e^2+r & e^2 \\ f^2 & f^2 & f^2+r \end{vmatrix}(d^2+e^2+f^2+r)$$

$$= r^2(d^2+e^2+f^2+r).$$

4. **(b) :** Let $f(x) = \dfrac{x^2}{x^3+200}, x \ge 1$

$$f'(x) = \frac{x(400-x^3)}{(x^3+200)^2}$$

$f'(x) > 0$ if $0 < x < \sqrt[3]{400}$ and $f'(x) < 0$ if $x > \sqrt[3]{400}$

$\Rightarrow$ $f(x)$ has a maximum at $x = \sqrt[3]{400},\ 7 < \sqrt[3]{400} < 8$

$\Rightarrow$ Either a_7 or a_8 is largest term.

$$a_7 = \frac{49}{543},\ a_8 = \frac{8}{89},\ a_7 > a_8 \therefore \text{ largest term} = \frac{49}{543}$$

5. **(a, b, c) :** $L = \lim\limits_{x\to 0} \dfrac{1}{x^3}\left(\dfrac{1}{\sqrt{1+x}} - \dfrac{1+ax}{1+bx}\right)$

$$= \lim_{x\to 0} \frac{1+bx-(1+ax)\sqrt{1+x}}{x^3} \lim_{x\to 0} \frac{1}{\sqrt{1+x}(1+bx)}$$

$$= \lim_{x\to 0} \frac{1+bx-(1+ax)(1+x)^{1/2}}{x^3}$$

$$= \lim_{x\to 0} \frac{1+bx-(1+ax)\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3+....\right)}{x^3}$$

$$= \lim_{x\to 0} \frac{\left(b-\frac{1}{2}-a\right)x+\left(-\frac{a}{2}+\frac{1}{8}\right)x^2+\left(\frac{a}{8}-\frac{1}{16}\right)x^3+...}{x^3}$$

Limit exists, if $b - \dfrac{1}{2} - a = 0$ and $-\dfrac{1}{8} + \dfrac{a}{2} = 0$

$\therefore a = \dfrac{1}{4}$ and $b = \dfrac{3}{4}$ $\qquad \therefore L = -\dfrac{1}{32}$

6. **(a, c) :** Given, $X \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$

The smallest set $X = \{1, 2, 3, 5, 9\} - \{1, 2\} = \{3, 5, 9\}$

The largest set $X = \{1, 2, 3, 5, 9\}$

7. **(0.093) :** The number of increasing functions is $\dbinom{6}{3} = 20$. $\therefore$ Required probability $= \dfrac{20}{6^3} = \dfrac{5}{54} = 0.093$

8. **(0.26) :** The number of non decreasing functions is $\dbinom{8}{3} = 56$.

$\therefore$ Required probability $= \dfrac{56}{6^3} = \dfrac{7}{27} = 0.26$

9. **(0.707) :** Let $G(x, y)$ is the centroid of triangle with vertices $P\left(\sqrt{2}\cos\theta, \sin\theta\right)$, $S(1, 0)$, $S_1(-1, 0)$, then

$3x = \sqrt{2}\cos\theta,\ 3y = \sin\theta$

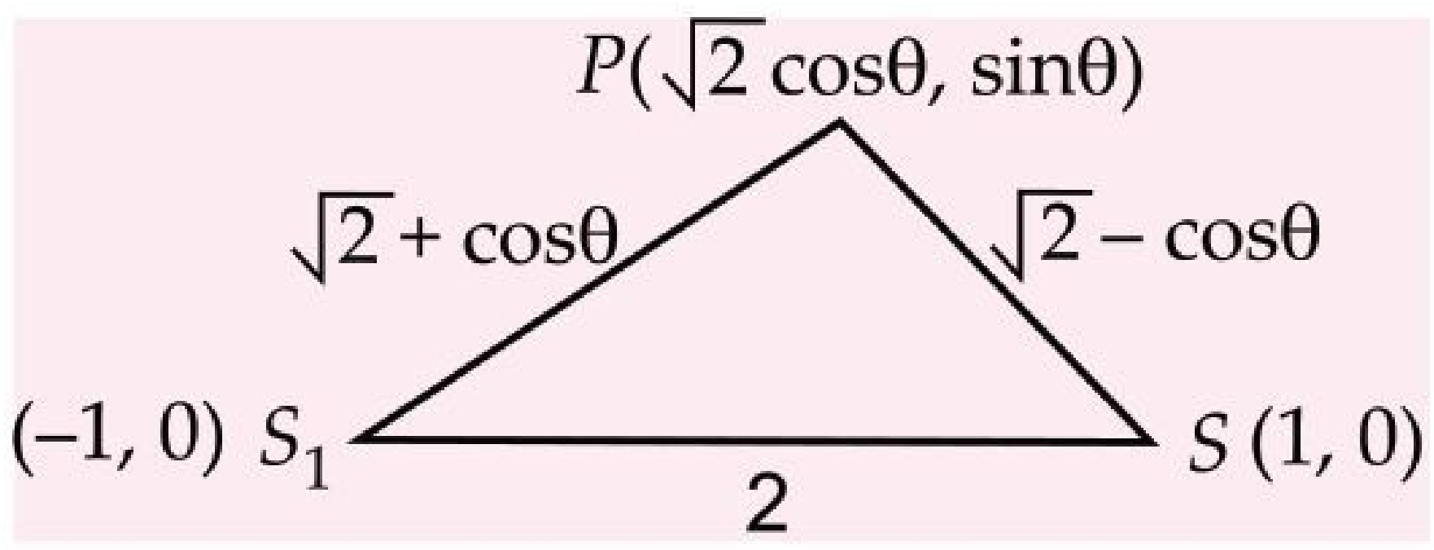
The locus of G is $\dfrac{x^2}{2} + y^2 = \dfrac{1}{9}$, eccentricity e is given by

$1 = 2(1 - e^2) \Rightarrow e = \dfrac{1}{\sqrt{2}} = 0.707$

10. **(2) :** $PS^2 = \left(\sqrt{2}\cos\theta - 1\right)^2 + \sin^2\theta = \left(\sqrt{2} - \cos\theta\right)^2$

$\therefore$ $PS = \sqrt{2} - \cos\theta,\ PS_1 = \sqrt{2} + \cos\theta$

$SS_1 = 2$



If $I(x, y)$ is the incentre, then

$$x = \frac{2\sqrt{2}\cos\theta + \sqrt{2} + \cos\theta - \sqrt{2} + \cos\theta}{2 + \sqrt{2} - \cos\theta + \sqrt{2} + \cos\theta} = \cos\theta$$

$$y = \frac{2\sin\theta}{2+2\sqrt{2}} = \left(\sqrt{2}-1\right)\sin\theta$$

The locus of I is $\dfrac{x^2}{1} + \dfrac{y^2}{\left(\sqrt{2}-1\right)^2} = 1$

Now, $\left(\sqrt{2}-1\right)^2 = 1 - e^2 \Rightarrow e = \sqrt{2\sqrt{2}-2}$.

11. **(20.87) :** $y(x) = x^2 + ax + b$, where $a = \int_0^1 y(t)\,dt$, $b = \int_0^1 t\,y(t)\,dt$

$$\therefore\ a = \frac{1}{3} + \frac{a}{2} + b,\ b = \frac{1}{4} + \frac{a}{3} + \frac{b}{2} \Rightarrow \frac{a}{2} - b = \frac{1}{3},\ \frac{b}{2} - \frac{a}{3} = \frac{1}{4}$$

Solving, $a = -5$, $b = -17/6$ $\therefore$ $f(x) = x^2 - 5x - 17/6$

$\Rightarrow$ $f'(x) = 2x - 5$

Tangent at $x = 0$ is $y + \dfrac{17}{6} = -5x$ which meets the x-axis at $\left(\dfrac{-17}{30}, 0\right)$

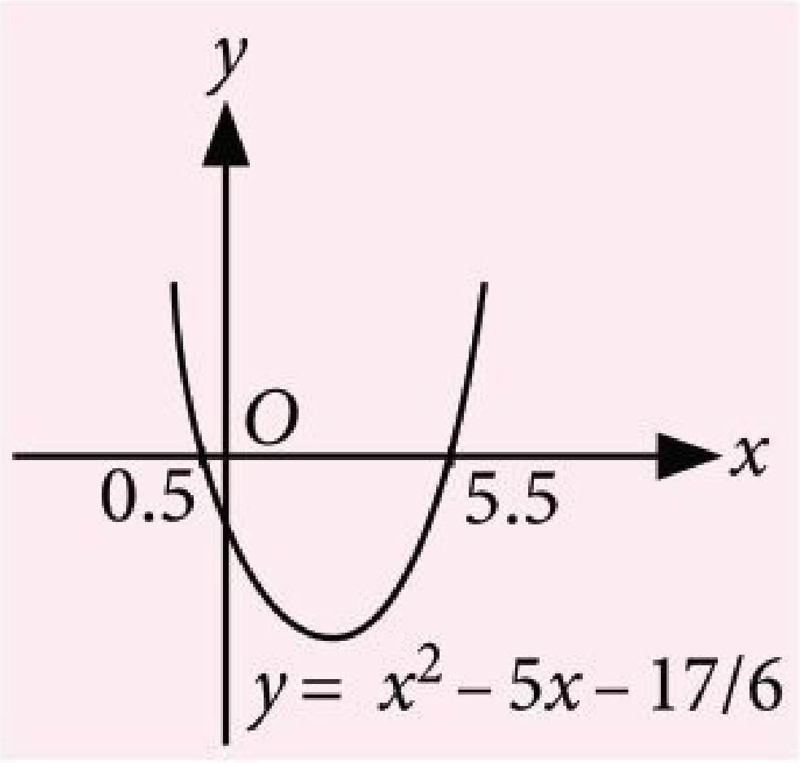
The normal $x = 0$ is $y+\frac{17}{6}=\frac{x}{5}$ which meets the x-axis at $\left(\frac{85}{6}, 0\right)$

$$\text{Area}=\frac{1}{2}\left(\frac{17}{6}\right)\left(\frac{85}{6}+\frac{17}{30}\right)=\frac{3757}{180}\;=20.87$$

12. (109) : $x^2-5x-\frac{17}{6}=0$ gives the roots

$$\alpha=\frac{5}{2}-\frac{1}{2}\sqrt{\frac{109}{3}},\ \beta=\frac{5}{2}+\frac{1}{2}\sqrt{\frac{109}{3}}$$

Required area, $A=-\int_{\alpha}^{\beta}f(x)\,dx$



$$=-\int_{\alpha}^{\beta}\left(x^2-5x-\frac{17}{6}\right)dx$$

$$=-\left[\frac{x^3}{3}-\frac{5x^2}{2}-\frac{17}{6}x\right]_{\alpha}^{\beta}$$

$$=\frac{-(\beta-\alpha)}{6}[2(\alpha^2+\beta^2+\alpha\beta)-15(\alpha+\beta)-17]$$

$$=\frac{-(\beta-\alpha)}{6}[2(\alpha+\beta)^2-2\alpha\beta-15(\alpha+\beta)-17]$$

But $\alpha + \beta = 5$, $\alpha\beta=\frac{-17}{6}$

$$\therefore\ A=\frac{-1}{6}\sqrt{\frac{109}{3}}\left[50+\frac{17}{3}-75-17\right]=\frac{109}{18}\sqrt{\frac{109}{3}}$$

13. (b) : Let $A'(x_2, y_2, z_2)$ be image of $A(2, 1, 6)$ about mirror $x + y - 2z = 3$, then

$$\frac{x_2-2}{1}=\frac{y_2-1}{1}=\frac{z_2-6}{-2}=\frac{-2(2+1-12-3)}{1^2+1^2+2^2}=4$$

$$\Rightarrow\ (x_2, y_2, z_2)=(6, 5, -2)$$

14. (c) : Let $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-6}{5}=\lambda$,

$\therefore$ $x = 2 + 3\lambda$, $y = 1 + 4\lambda$, $z = 6 + 5\lambda$, lies on plane $x + y - 2z = 3$

$\therefore$ $2 + 3\lambda + 1 + 4\lambda - 2(6 + 5\lambda) = 3$

$\Rightarrow$ $3 + 7\lambda - 12 - 10\lambda = 3 \Rightarrow -3\lambda = 12 \Rightarrow \lambda = -4$

$\therefore$ Point $B \equiv (-10, -15, -14)$.

(15-16) : 15. (b) 16. (d)

Equation of tangent to $y^2 = 8x$ is $yt - x - 2t^2 = 0$ ...(1)

Equation of tangent to ellipse is $\frac{x\cos\theta}{2\sqrt{\alpha}}+\frac{y\sin\theta}{\sqrt{\alpha}}=1$ ...(2)

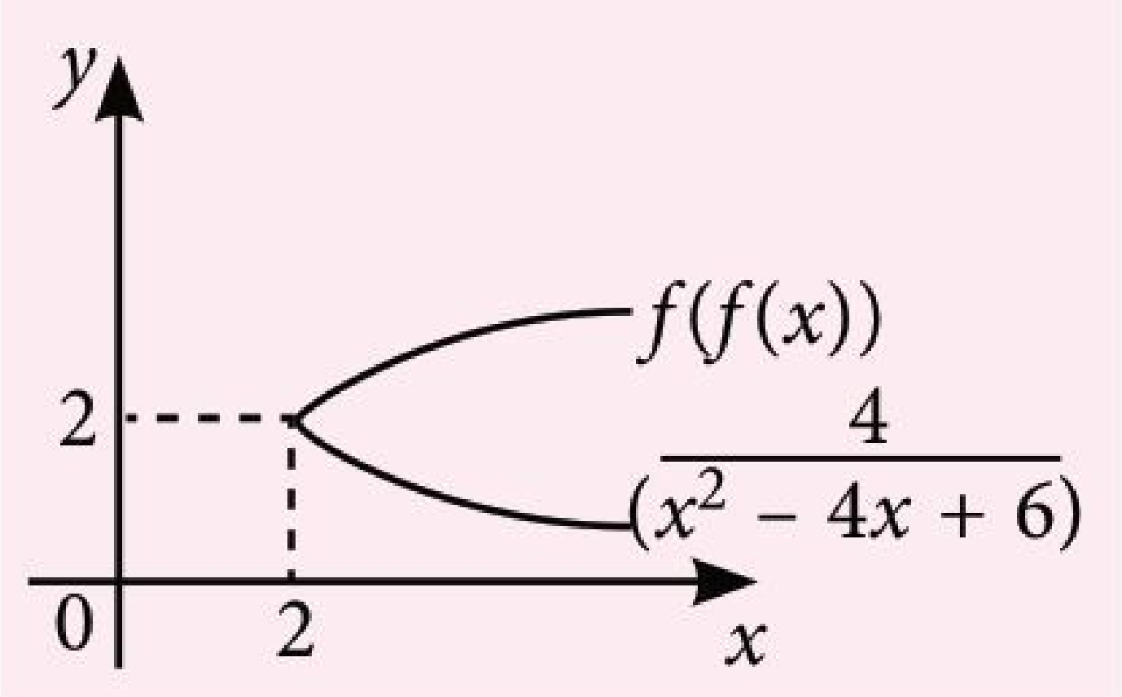
If the tangent meets the coordinate axes at A and B then A is $\left(\frac{2\sqrt{\alpha}}{\cos\theta}, 0\right)$ and B is $\left(0, \frac{\sqrt{\alpha}}{\sin\theta}\right)$.

Let mid point of AB is (h, k) then,

$$h=\frac{\sqrt{\alpha}}{\cos\theta}, k=\frac{\sqrt{\alpha}}{2\sin\theta}, h=-t^2, k=t\Rightarrow k^2=-h \text{ or } y^2=-x$$

$$k^2=-h\Rightarrow\ \frac{\alpha}{\sin^2\theta}=\frac{-4\sqrt{\alpha}}{\cos\theta}\ \Rightarrow\ \sqrt{\alpha}=-\frac{4\sin^2\theta}{\cos\theta}=6$$

17. (1) : $f(f(x))$ is also an increasing function



Let $y=\frac{4}{(x^2-4x+6)}\ \Rightarrow\ \frac{dy}{dx}=-\frac{8(x-2)}{(x^2-4x+6)^2}$

is decreasing function.

Only one solution is possible.

18. (1) : $\Delta=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ \alpha & e^{\alpha} & 1\\ \sqrt{\alpha^2+e^{2\alpha}} & 0 & 1\end{vmatrix}$

$\Delta=\frac{1}{2}\left|e^{\alpha}\sqrt{\alpha^2+e^{2\alpha}}\right|\ ;\ \frac{d\Delta}{d\alpha}>0$

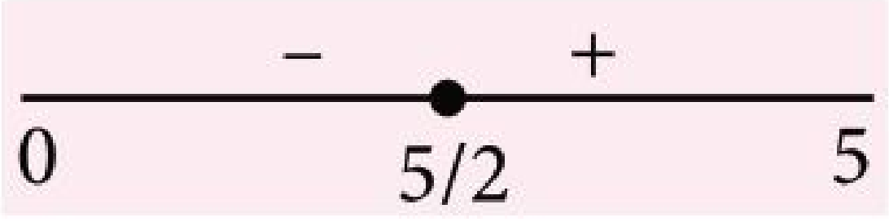
$\Rightarrow \Delta$ is increasing function.

$\Delta_{\max}=\sqrt{(\ln 2)^2+4}$

$\lambda = 1$

19. (3) : $f(x)=\begin{cases}\frac{1}{x+5}+\frac{1}{x}; & x\ge 5\\ \frac{1}{x+5}+\frac{1}{10-x}; & 0\le x<5\\ \frac{1}{5-x}+\frac{1}{10-x}; & x<0\end{cases}$

$f'(x)=\begin{cases}-\frac{1}{(x+5)^2}-\frac{1}{x^2}; & x\ge 5\\ -\frac{1}{(x+5)^2}+\frac{1}{(10-x)^2}; & 0<x<5\\ \frac{1}{(5-x)^2}+\frac{1}{(10-x)^2}; & x<0\end{cases}$

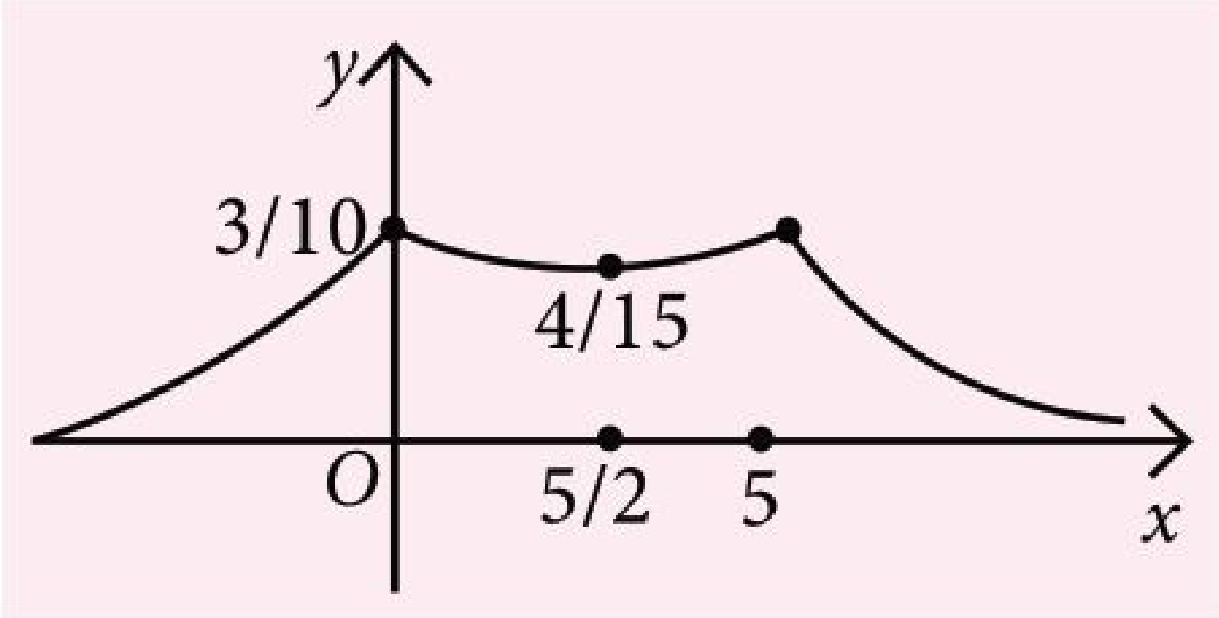


$\Rightarrow$ $0 < x < 5$ and

$(x + 5)^2 - (10 - x)^2 = 0$

$\Rightarrow x = 5/2$

Graph of $f(x)$



$\Rightarrow$ Absolute maximum value is 3/10

$10A = 3$

* * *

CUET (UG)

PRACTICE PAPER 2022

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Max. Marks : 200 Time : 45 minutes

Multiple Choice Questions (MCQs)

1. If $f : N \to N$, where $f(x) = x - (-1)^x$, then f is
(a) One-one and into (b) Many-one and into
(c) One-one and onto (d) Many-one and onto

2. $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$ is equal to
(a) $\sqrt{\dfrac{x^2+2}{x^2+3}}$ (b) $\sqrt{\dfrac{x^2+2}{x^2+1}}$
(c) $\sqrt{\dfrac{x^2+1}{x^2+2}}$ (d) None of these

3. Find the value of $\int 2^{2^{2^x}} 2^{2^x} 2^x \, dx$.
(a) $\dfrac{1}{(\log 2)^3} 2^{2^{2^x}} + C$ (b) $\dfrac{1}{(\log 2)^3} 2^{2^x} + C$
(c) $\dfrac{1}{(\log 2)^2} 2^{2^x} + C$ (d) $\dfrac{1}{(\log 2)^4} 2^{2^{2^x}} + C$

4. The function $f(x) = \cot^{-1}x + x$ increases in the interval
(a) $(1, \infty)$ (b) $(-1, \infty)$
(c) $(0, \infty)$ (d) $(-\infty, \infty)$

5. If the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{ax+3}{2y+f}$ represents a circle, then the value of 'a' is
(a) 2 (b) – 2 (c) 3 (d) – 4

6. Find x, y, z and w respectively such that

$$\begin{bmatrix} x-y & 2z+w \\ 2x-y & 2x+w \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 12 & 15 \end{bmatrix}.$$

(a) 7, 2 , 1, 1 (b) 7, 5, 3, 8
(c) 1, 2, 5, 6 (d) 6, 3, 2, 1

7. The angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+4\hat{k}$ is
(a) 90° (b) 45° (c) 30° (d) 15°

8. If $\Delta = \begin{vmatrix} a+1 & a+2 & a+4 \\ a+3 & a+5 & a+8 \\ a+7 & a+10 & a+14 \end{vmatrix}$, then value of Δ is
(a) 2 (b) – 2 (c) 4 (d) – 4

9. The angle between the planes $x + y = 0$ and $y - z = 1$ is
(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

10. If $f(x) = \dfrac{\sqrt{4+x}-2}{x}$, $x \neq 0$ be continuous at $x = 0$, then $f(0) =$
(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) 2 (d) $\dfrac{3}{2}$

11. The area of the region enclosed by the lines $y = x$, $x = e$ and curve $y = \dfrac{1}{x}$ and the positive x-axis is
(a) 1 sq. unit (b) $\dfrac{3}{2}$ sq. units
(c) $\dfrac{5}{2}$ sq. units (d) $\dfrac{1}{2}$ sq. unit

12. Find cofactors of a_{21} and a_{31} of the matrix

$$A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}.$$

(a) –16, 8 (b) –16, –8 (c) 16, 8 (d) 16, –8

13. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.
(a) $\dfrac{1}{4}$ (b) $\dfrac{3}{5}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{2}{5}$

14. The point on the curve $y = (x-3)^2$ where the tangent is parallel to the chord joining (3, 0) and (4, 1) is
(a) $\left(-\dfrac{7}{2}, \dfrac{1}{4}\right)$ (b) $\left(\dfrac{5}{2}, \dfrac{1}{4}\right)$
(c) $\left(-\dfrac{5}{2}, \dfrac{1}{4}\right)$ (d) $\left(\dfrac{7}{2}, \dfrac{1}{4}\right)$

15. If A and B are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then find $P(\text{not } A \text{ and not } B)$.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

16. $\int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx =$

(a) $\frac{2}{\sqrt{\tan x}} - \frac{2}{3}(\tan x)^{3/2} + C$

(b) $-\frac{2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + C$

(c) $-\frac{2}{\sqrt{\tan x}} - \frac{2}{3}(\tan x)^{2/3} + C$

(d) None of these

17. If $f: [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals

(a) $\frac{x + \sqrt{x^2 - 4}}{2}$ (b) $\frac{x}{1 + x^2}$

(c) $\frac{x - \sqrt{x^2 - 4}}{2}$ (d) $1 + \sqrt{x^2 - 4}$

18. The solution of $\cos(x + y)\, dy = dx$ is

(a) $y = \tan\left(\frac{x + y}{2}\right) + C$

(b) $y = \cos^{-1}\left(\frac{y}{x}\right) + C$

(c) $y = x \sec\left(\frac{y}{x}\right) + C$

(d) none of these

19. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, then $A^4 - 2^4(A - I) =$

(a) $I - A$ (b) $2I - A$ (c) $I + A$ (d) $A - 2I$

20. Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is

(a) $\frac{1}{4\sqrt{x}} \sec\sqrt{x} \sin\sqrt{x}$

(b) $\frac{1}{4\sqrt{x}} (\sec\sqrt{x})^{3/2} \cdot \sin\sqrt{x}$

(c) $\frac{1}{2}\sqrt{x} \sec\sqrt{x} \sin\sqrt{x}$

(d) $\frac{1}{2}\sqrt{x}(\sec\sqrt{x})^{3/2} \cdot \sin\sqrt{x}$

21. If $\overrightarrow{AB} \times \overrightarrow{AC} = 2\hat{i} - 4\hat{j} + 4\hat{k}$, then the area of ΔABC is

(a) 3 sq. units (b) 4 sq. units

(c) 16 sq. units (d) 9 sq. units

22. The number of positive integral solutions of $\tan^{-1} x + \cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ is

(a) 0 (b) 1 (c) 2 (d) > 2

23. Find the minor of a_{22} of the matrix

$$\begin{bmatrix} 1 & 6 & 1 \\ 5 & 3 & 0 \\ 2 & 2 & 9 \end{bmatrix}.$$

(a) 5 (b) 6 (c) 7 (d) 8

24. If there is an error of a% in measuring the edge of a cube, then percentage error in its surface area is

(a) $2a$% (b) $\frac{a}{2}$%

(c) $3a$% (d) None of these

25. The value of $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ is

(a) $3\sqrt{x} + 3(\sqrt[3]{x}) - 6\sqrt[6]{x} + \log(\sqrt[6]{x} + 1) + C$

(b) $2\sqrt{x} + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x} + 1) + C$

(c) $2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x} + 1) + C$

(d) None of the above

26. The number of commutative binary operations that can be defined on a set of 2 elements is

(a) 8 (b) 6 (c) 4 (d) 2

27. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{8}$

28. If $f(x) = x^n$, then the value of

$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots.. + \frac{(-1)^n f^n(1)}{n!}$ is

(a) $2n$ (b) 2^{n-1} (c) 0 (d) 1

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Physics, Chemistry, Mathematics & Biology
Section-III: General Test
COMPUTER BASED TEST

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29. If $|\vec{a}|=5, |\vec{b}|=13$ and $|\vec{a}\times\vec{b}|=25$, find $\vec{a}\cdot\vec{b}$.

(a) ±10 (b) ±40 (c) ±60 (d) ±25

30. The solution of $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{\sqrt{1+x^2}}$ is

(a) $y=\frac{1+x^2}{x}+\frac{c}{x}$ (b) $y=\frac{\sqrt{1+x^2}}{x}+\frac{c}{x}$

(c) $y=\frac{x}{\sqrt{1+x^2}}+cx$ (d) none of these

31. For any square matrix A, AA^T is a

(a) unit matrix (b) symmetric matrix

(c) skew-symmetric matrix

(d) diagonal matrix

32. The distance of the origin from the plane through the points (1, 1, 0), (1, 2, 1) and (–2, 2, –1) is

(a) $\frac{3}{\sqrt{11}}$ (b) $\frac{5}{\sqrt{22}}$ (c) 3 (d) $\frac{4}{\sqrt{22}}$

33. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then

$x^4+y^4+z^4+4x^2y^2z^2=$

(a) $x^2+y^2+y^2z^2$ (b) $2(x^2y^2+y^2z^2+z^2x^2)$

(c) $(x+y)^2$ (d) $(x+y+z)^2$

34. $\int_0^{\pi/2}\frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) None of these

Assertion & Reasoning Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

(c) If Statement-I is true but Statement-II is false.

(d) If Statement-I is false and Statement-II is true.

35. Let A be a 2 × 2 matrix.

Statement-I : adj(adj A) = A.

Statement-II: |adj A| = $|A|$.

36. **Statement-I :** If $f'(x)=(x-1)^3(x-2)^8$, then $f(x)$ has neither maximum nor minimum at $x=2$.

Statement-II : $f'(x)$ changes sign from negative to positive at $x=2$.

37. **Statement-I :** The area bounded by the parabola $y^2=4ax$ and the line $x=a$ and $x=4a$ is $\frac{56a^2}{3}$ sq. units.

Statement-II : The area bounded by the curves $y=3x$ and $y=x^2$ is 9.5 sq. units.

38. **Statement-I :** The equation of a plane which passes through (2, –3, 1) and normal to the line joining the points (3, 4, –1) and (2, –1, 5) is given by $x+5y-6z+19=0$.

Statement-II : The length of perpendicular from the point (7, 14, 5) to the plane $2x+4y-z=2$ is $2\sqrt{21}$ units.

Match the Following

39. Match the following and choose correct option from given below.

	Column I		**Column II**
(P)	If $y=\log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}=$	(1)	$1/\sqrt{x^2+1}$
(Q)	$\frac{d}{dx}\{\log(x+\sqrt{x^2+1})\}=$	(2)	0
(R)	If $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$, then $\frac{dy}{dx}=$	(3)	$-4x/(1-x^4)$
(S)	If $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)-\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $x\in(-1,0)$, then $\frac{dy}{dx}=$	(4)	$\frac{4e^{2x}}{(e^{2x}+1)^2}$

	P	Q	R	S		P	Q	R	S
(a)	3	2	4	1	(b)	3	1	4	2
(c)	2	3	1	4	(d)	1	4	2	3

40. Match the following.

	Column I		Column II
(P)	$\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=$	(1)	$\tan^{-1}\frac{4}{3}-x$
(Q)	$\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$, where $x\in\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$ is equal to	(2)	$\frac{\pi}{4}$
(R)	$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, where $x\in\left(0,\frac{\pi}{4}\right)$ is equal to	(3)	$\cos^{-1}\left(\frac{33}{65}\right)$
(S)	$\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)=$	(4)	$\frac{x}{2}$

	P	Q	R	S		P	Q	R	S
(a)	4	2	3	1	(b)	3	1	2	4
(c)	3	1	4	2	(d)	1	3	2	4

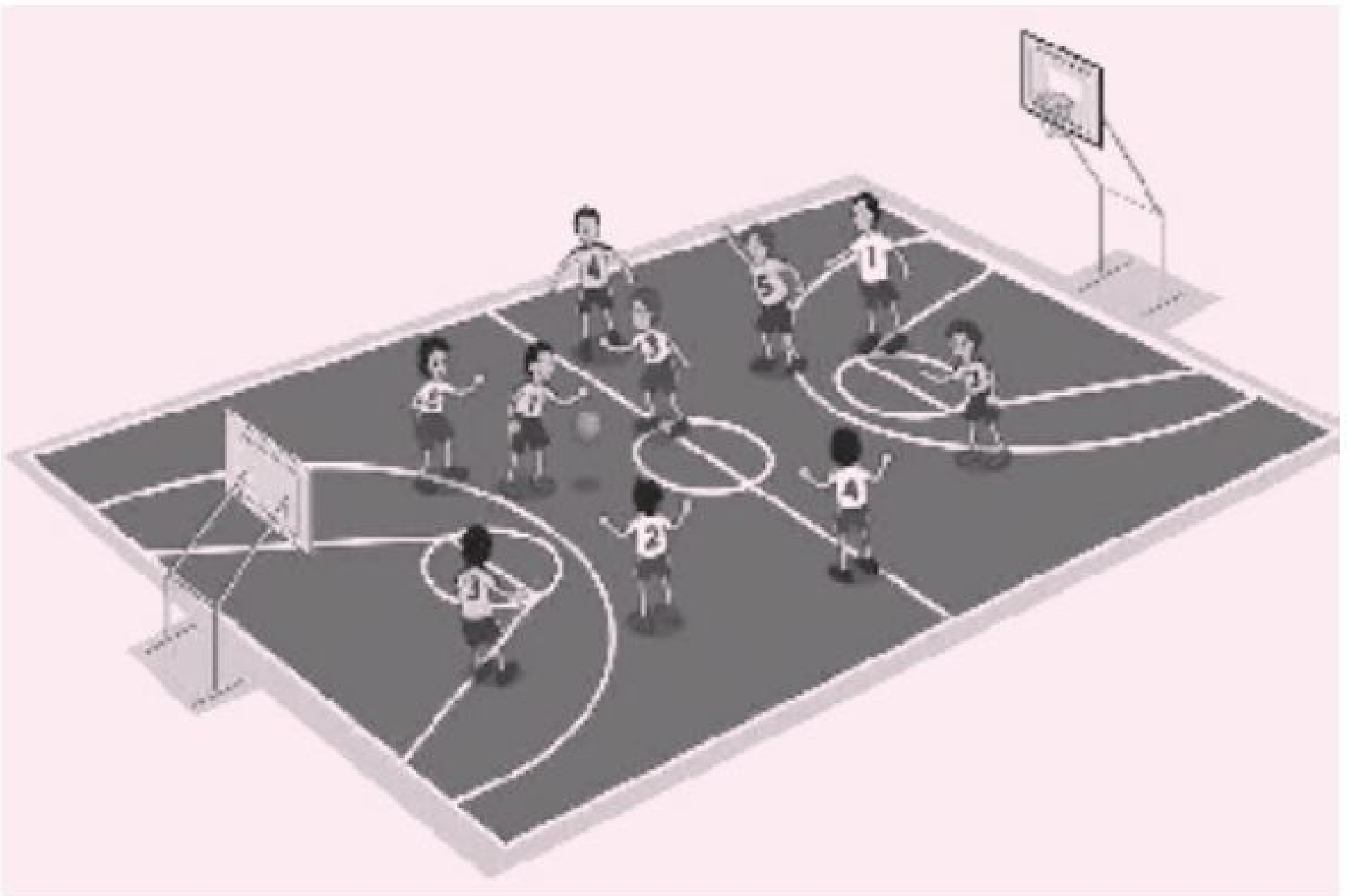
Case Based MCQs

Case I : Read the passage given below and answer the following questions from 41 to 45.

Between students of class XII of two schools A and B, basketball match is organised. For which, a team from each school is chosen, say T_1 be the team of school A and T_2 be the team of school B. These teams have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probability of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2},\frac{3}{10}$ and $\frac{1}{5}$ respectively.

Each team gets 2 points for a win, 1 point for a draw and 0 point for a loss in a game.

Let X and Y denote the total points scored by team T_1 and T_2 respectively, after two games.



41. $P(T_2$ winning a match against $T_1)$ is equal to
(a) 1/5 (b) 1/6
(c) 1/3 (d) none of these

42. $P(T_2$ drawing a match against $T_1)$ is equal to
(a) 1/2 (b) 1/3 (c) 1/6 (d) 3/10

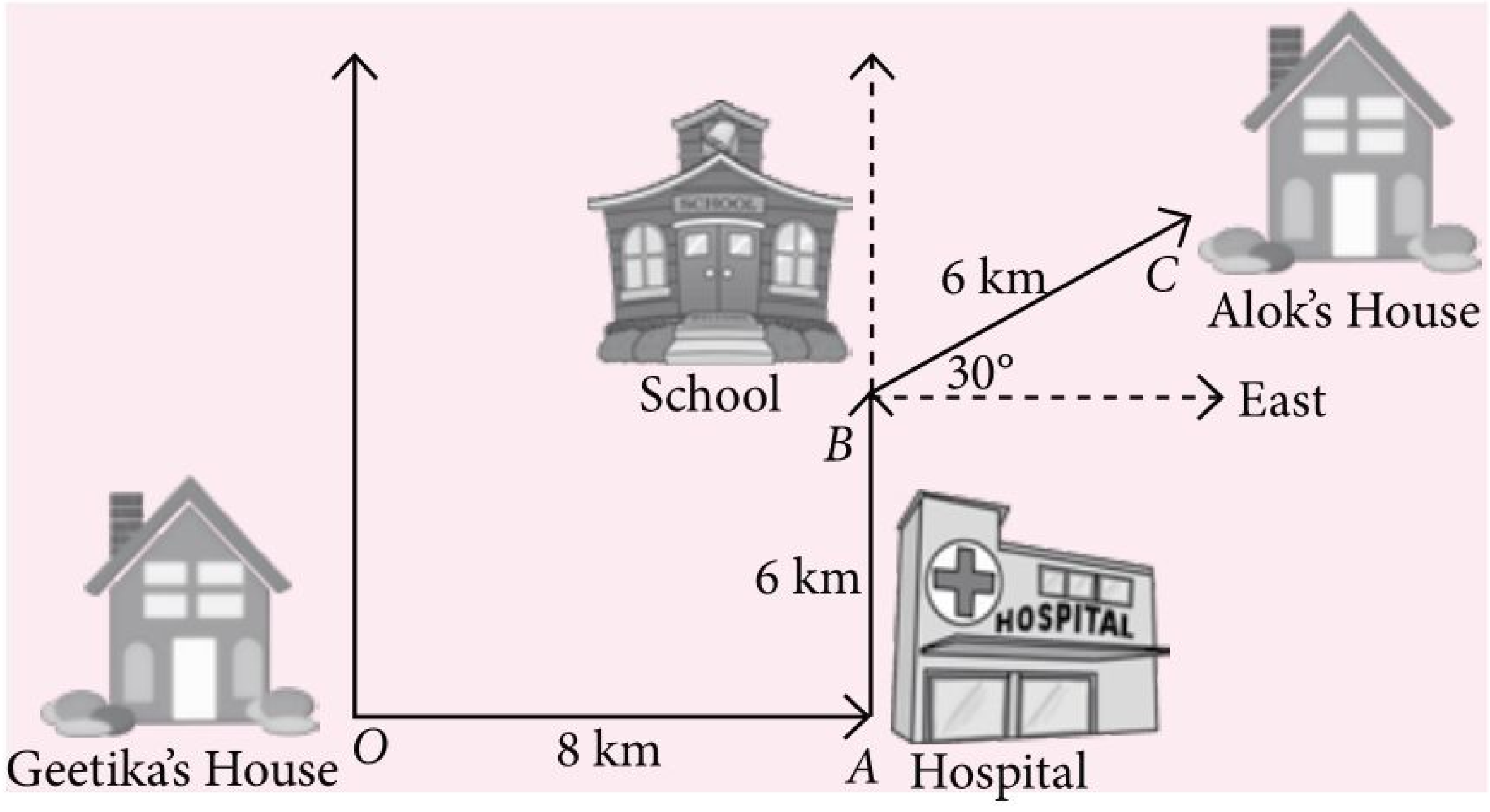
43. $P(X > Y)$ is equal to
(a) 1/4 (b) 5/12 (c) 1/20 (d) 11/20

44. $P(X = Y)$ is equal to
(a) 11/100 (b) 1/3 (c) 29/100 (d) 1/2

45. $P(X + Y = 8)$ is equal to
(a) 0 (b) 5/12 (c) 13/36 (d) 7/12

Case II : Read the passage given below and answer the following questions from 46 to 50.

Geetika's house is situated at Shalimar Bagh at point O, for going to Alok's house she first travels 8 km by bus in the East. Here at point A, a hospital is situated. From Hospital, Geetika takes an auto and goes 6 km in the North, here at point B school is situated. From school, she travels by bus to reach Alok's house at point C which is at 30° East, 6 km from the school.



46. What is the vector distance between Geetika's house and school?
(a) $8\hat{i}-6\hat{j}$ (b) $8\hat{i}+6\hat{j}$ (c) $8\hat{i}$ (d) $6\hat{j}$

47. How much distance Geetika travels to reach school?
(a) 14 km (b) 15 km (c) 16 km (d) 17 km

48. What is the vector distance from school to Alok's house?
(a) $\sqrt{3}\hat{i}+\hat{j}$ (b) $3\sqrt{3}\hat{i}+3\hat{j}$
(c) $6\hat{i}$ (d) $6\hat{j}$

49. What is the vector distance from Geetika's house to Alok's house?
(a) $(8+3\sqrt{3})\hat{i}+9\hat{j}$ (b) $4\hat{i}+6\hat{j}$
(c) $15\hat{i}$ (d) $16\hat{j}$

50. What is the total distance travelled by Geetika from her house to Alok's house?
(a) 19 km (b) 20 km (c) 21 km (d) 22 km

SOLUTIONS

1. **(c) :** $f(x)=\begin{cases} x-1, & x \text{ is even} \\ x+1, & x \text{ is odd} \end{cases}$

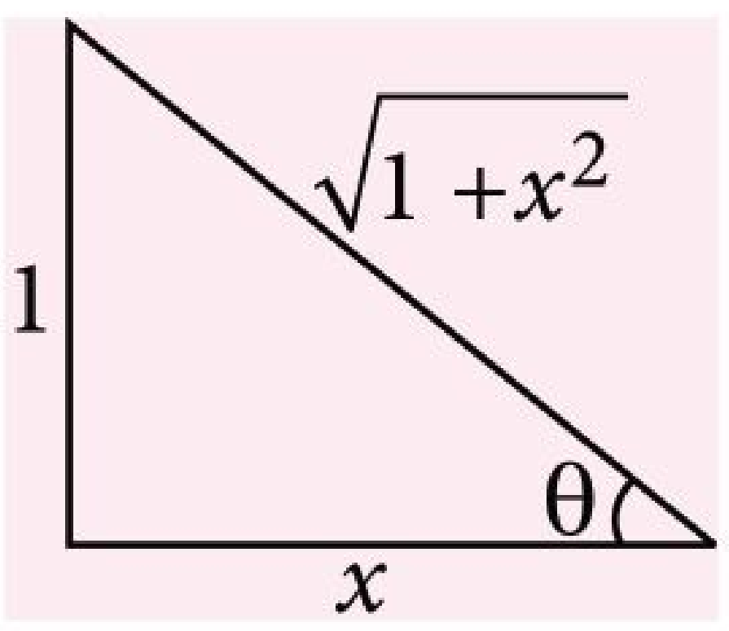
which is clearly one-one and onto.

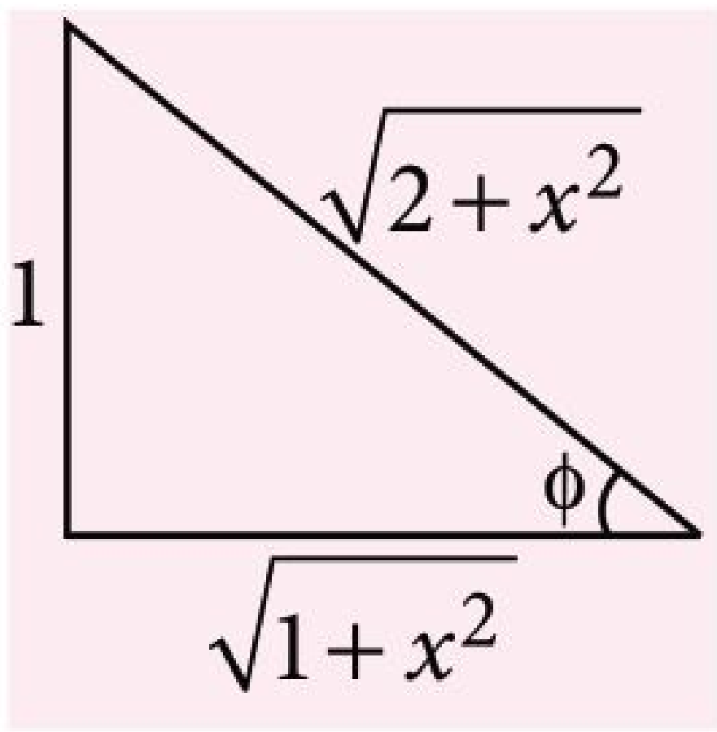
2. **(c) :** We have, $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

Let $\cot^{-1}x=\theta \Rightarrow \cot\theta = x$

$$\Rightarrow \quad \sin\theta=\frac{1}{\sqrt{1+x^2}}$$

$$\therefore \quad \cos[\tan^{-1}\{\sin\theta\}]=\cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$





Again, let $\tan^{-1}\frac{1}{\sqrt{1+x^2}}=\phi \Rightarrow \tan\phi=\frac{1}{\sqrt{1+x^2}}$

$$\Rightarrow \quad \cos\phi=\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

$$\therefore \quad \cos\left[\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right]=\cos\phi \;=\sqrt{\frac{x^2+1}{x^2+2}}$$

3. **(a) :** Let $I=\int 2^{2^{2^x}}\,2^{2^x}\,2^x\,dx$

Putting $2^{2^{2^x}}=t \Rightarrow 2^{2^{2^x}}\,2^{2^x}\,2^x(\log 2)^3\,dx=dt$, we get

$$I=\int\frac{1}{(\log 2)^3}dt=\frac{1}{(\log 2)^3}t+C=\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$$

4. **(d):** $f(x) = \cot^{-1}x + x$

$$\Rightarrow \quad f'(x)=\frac{-1}{1+x^2}+1 \;\Rightarrow\; f'(x)=\frac{x^2}{1+x^2}\geq 0,\;\forall\, x\in R$$

$\therefore$ $f(x)$ is increasing in $(-\infty, \infty)$.

5. **(b):** We have, $\frac{dy}{dx}=\frac{ax+3}{2y+f}$

$\Rightarrow (ax+3)dx=(2y+f)dy$

$\Rightarrow a\frac{x^2}{2}+3x=y^2+fy+C$ (Integrating)

$\Rightarrow -\frac{a}{2}x^2+y^2-3x+fy+C=0$

This will represent a circle, if $\frac{-a}{2}=1\Rightarrow a=-2$

[$\because$ coefficient of x^2 = coefficient of y^2]

6. **(a)**

7. **(a) :** Given that, $\vec{a}=\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+4\hat{k}$

$\therefore$ $\vec{a}+\vec{b}=2\hat{i}+8\hat{k}$ and $\vec{a}-\vec{b}=2\hat{j}$

Let θ be the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, then

$$\cos\theta=\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$$

$$=\frac{(2\hat{i}+0\hat{j}+8\hat{k})\cdot(0\hat{i}+2\hat{j}+0\hat{k})}{\sqrt{2^2+0^2+8^2}\sqrt{0^2+2^2+0^2}}$$

$$=\frac{0+0+0}{\sqrt{4+64}\sqrt{4}}=0$$

$$\Rightarrow \cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}=90^\circ$$

8. **(b):** Applying $C_2\to C_2-C_1$, $C_3\to C_3-C_1$, we get

$$\Delta=\begin{vmatrix} a+1 & 1 & 3 \\ a+3 & 2 & 5 \\ a+7 & 3 & 7 \end{vmatrix}$$

Applying $R_2\to R_2-R_1$, $R_3\to R_3-R_1$, we get

$$\Delta=\begin{vmatrix} a+1 & 1 & 3 \\ 2 & 1 & 2 \\ 6 & 2 & 4 \end{vmatrix}=0-(8-12)+3(4-6)=-2$$

9. **(c) :** The DR's of the normals to the gives planes are (1, 1, 0) and (0, 1, –1).

Now, the angle between the planes is

$$\theta=\cos^{-1}\left|\frac{1\times 0+1\times 1-0\times 1}{\sqrt{2}\cdot\sqrt{2}}\right|=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

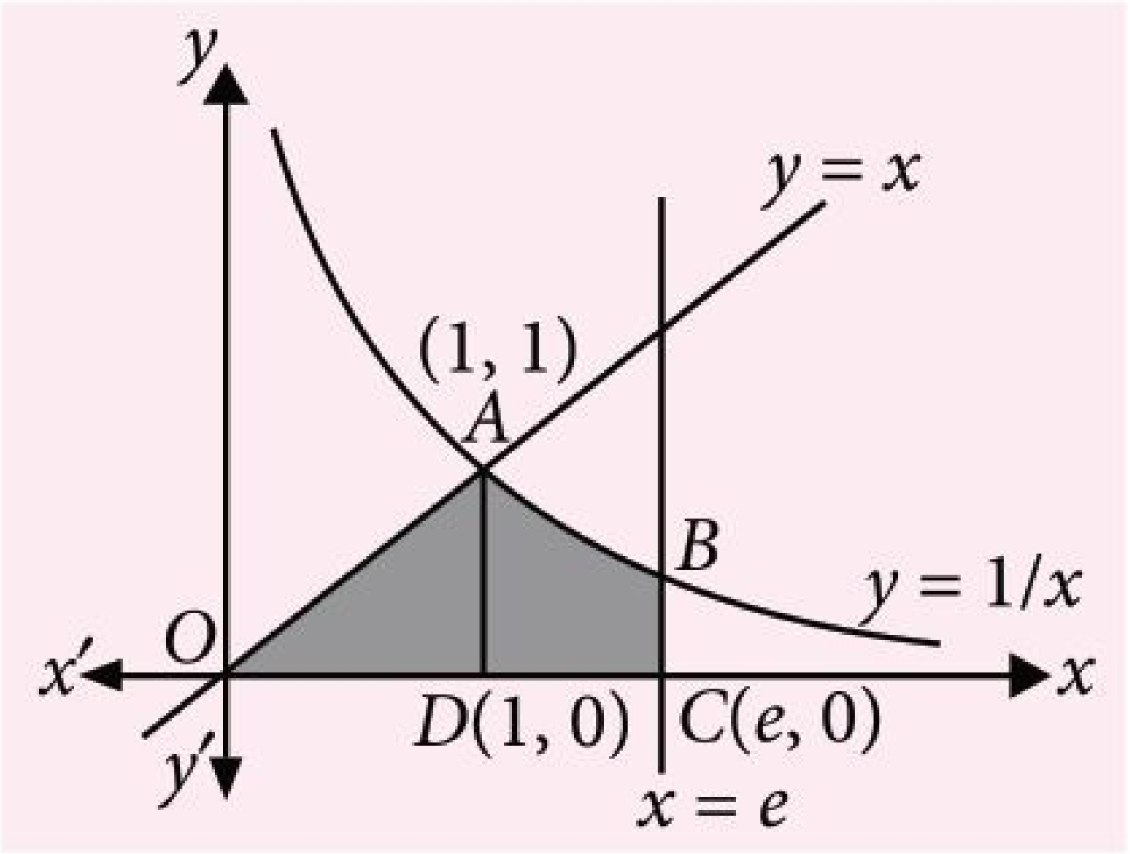
10. **(b):** $f(0)=\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\frac{\sqrt{4+x}-2}{x}$

$$=\lim_{x\to 0}\left(\frac{\sqrt{4+x}-2}{x}\times\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)$$

$$=\lim_{x\to 0}\frac{4+x-4}{x(\sqrt{4+x}+2)}=\lim_{x\to 0}\frac{x}{x(\sqrt{4+x}+2)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}=\frac{1}{2+2}=\frac{1}{4}.$$

11. (b): We have, $y = x, x = e, y = \frac{1}{x}$ and $x \geq 0$



$\therefore$ Required area = Area of ΔODA + Area of region $ABCD$

$$=\frac{1}{2}(1\times 1)+\int_1^e \frac{1}{x}dx = \frac{1}{2}+\left[\log|x|\right]_1^e = \frac{1}{2}+1=\frac{3}{2} \text{ sq. units}$$

12. (a): Let M_{ij} and c_{ij} respectively denote the minor and cofactor of element a_{ij} in A. Then,

$$M_{21}=\begin{vmatrix}3 & -2\\ 5 & 2\end{vmatrix}=6+10=16 \Rightarrow c_{21}=-M_{21}=-16$$

$$M_{31}=\begin{vmatrix}3 & -2\\ -5 & 6\end{vmatrix}=18-10=8 \Rightarrow c_{31}=M_{31}=8$$

13. (d): We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\, P(B)$

(Since A, B are independent)

Thus, we have

$1/4 + P(B) - (1/4)\, P(B) = 2P(B) - 1/4$

$\Rightarrow P(B) = 2/5.$

14. (d): We have, $y = (x - 3)^2$

Now, slope $=\frac{dy}{dx}=2(x-3)$;

Slope of chord joining (3, 0) and (4, 1) $=\frac{1-0}{4-3}=1$

For parallel lines, slopes to be equal

$\therefore 2(x - 3) = 1 \Rightarrow x=\frac{7}{2}$ and $y=\left(\frac{7}{2}-3\right)^2=\frac{1}{4}$

Hence, point is $\left(\frac{7}{2},\frac{1}{4}\right)$.

15. (b): Here $P(A)\cdot P(B) = \frac{1}{3}\times\frac{1}{4}=\frac{1}{12}=P(A\cap B)$

$\Rightarrow$ Events A and B are independent.

$\Rightarrow$ Events $\bar{A}$ and $\bar{B}$ are also independent.

Now $P(\bar{A}\cap\bar{B})=P(\bar{A})\cdot P(\bar{B})$

($\because$ $\bar{A}$ and $\bar{B}$ are independent events)

$= (1 - P(A))(1 - P(B))$

$$=\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$$

16. (b): Let $I=\int\frac{1}{\sqrt{\sin^3 x\cos^5 x}}dx$

$$=\int\frac{1}{\sin^{3/2}x\cos^{5/2}x}dx=\int\frac{\sec^4 x}{\tan^{3/2}x}dx$$

[Dividing numerator and denominator by $\cos^4 x$]

$$=\int\frac{(1+\tan^2 x)}{\tan^{3/2}x}\sec^2 x\, dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$, we get

$$I=\int\frac{1+t^2}{t^{3/2}}\,dt=\int(t^{-3/2}+t^{1/2})\,dt=\frac{-2}{\sqrt{t}}+\frac{t^{3/2}}{3/2}+C$$

$$=-\frac{2}{\sqrt{\tan x}}+\frac{2}{3}(\tan x)^{3/2}+C$$

17. (a): Let $y=f(x)=x+\frac{1}{x} \Rightarrow x^2 - xy + 1 = 0$

$$\Rightarrow x=\frac{y\pm\sqrt{y^2-4}}{2}$$

$\because y\in[2,\infty) \Rightarrow x\in[1,\infty)$

Hence, $f^{-1}(x)=\frac{x+\sqrt{x^2-4}}{2}$

18. (a): Given differential equation is $\cos(x + y)\, dy = dx$

$$\Rightarrow \frac{dy}{dx}=\sec(x+y)$$

Substitute $x + y = t \Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$

Hence, given differential equation becomes

$$\frac{dt}{dx}-1=\sec t \Rightarrow \frac{dt}{dx}=1+\sec t=\frac{1+\cos t}{\cos t}$$

$$\Rightarrow \frac{\cos t}{1+\cos t}dt=dx \Rightarrow \int\left(1-\frac{1}{1+\cos t}\right)dt=x+C$$

$$\Rightarrow t-\frac{1}{2}\int\sec^2\left(\frac{t}{2}\right)dt=x+C \Rightarrow t-\tan\left(\frac{t}{2}\right)=x+C$$

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(a)	2.	(c)	3.	(b)	4.	(c)	5.	(d)
6.	(b)	7.	(b,d)	8.	(a,c)	9.	(c)	10.	(a,b,c)
11.	(a,b,c)	12.	(b,c)	13.	(b)	14.	(b)	15.	(c)
16.	(a)	17.	(8)	18.	(6)	19.	(2)	20.	(2)

$$\Rightarrow\ x + y - \tan\left(\frac{x+y}{2}\right) = x + C \Rightarrow y = \tan\left(\frac{x+y}{2}\right) + C$$

19. (b): We have, $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix}$$

Now, $A^4 - 2^4(A - I) = \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix} - 16\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix} - \begin{bmatrix} 0 & 16 \\ 0 & 16 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Also, $2I - A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$\therefore\ A^4 - 2^4(A - I) = 2I - A$

20. (b): Let $y = \sqrt{\sec\sqrt{x}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sec\sqrt{x}}} \cdot \sec\sqrt{x} \cdot \tan\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{1/2}\frac{\sin\sqrt{x}}{\cos\sqrt{x}} = \frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2} \cdot \sin\sqrt{x}$$

21. (a): Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}|2\hat{i} - 4\hat{j} + 4\hat{k}|$

$$= \frac{1}{2}\left[\sqrt{(2)^2 + (-4)^2 + (4)^2}\right] = 3 \text{ sq. units}$$

22. (c): $\tan^{-1}x + \tan^{-1}\frac{1}{y} = \tan^{-1}3$

$\Rightarrow\ x + \frac{1}{y} = 3\left(1 - \frac{x}{y}\right)$ provided, $\frac{x}{y} < 1$, *i.e.*, $x < y$.

$\Rightarrow\ xy + 1 = 3(y - x)$.

$\Rightarrow\ y = \frac{1+3x}{3-x} \Rightarrow x = 1,\ y = 2;\ x = 2, y = 7$

23. (c): $M_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 9 \end{vmatrix} = 9 - 2 = 7$

24. (a): Let edge of cube be x cm

$\therefore\ \frac{\Delta x}{x} \times 100 = a$ (Given)

Now, $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$

So, $\Delta S = \frac{dS}{dx} \times \Delta x$

$$\Rightarrow\ \frac{\Delta S}{S} \times 100 = \frac{12x}{6x^2} \times \frac{ax}{100} \times 100 = 2a\%$$

25. (c): Let $I = \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

Putting $x = t^6 \Rightarrow dx = 6t^5\,dt$, we get

$$I = \int \frac{6t^5}{t^3 + t^2}\,dt = 6\int \frac{t^3}{t+1}\,dt = 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right)dt$$

$= 2t^3 - 3t^2 + 6t - 6\log(t + 1) + C$

$= 2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x} + 1) + C$

26. (d): Required no. of operations $= (2)^{2(2-1)/2} = 2$

27. (b): Let E_1, E_2 and A be the events defined as follows:
E_1 : die shows six *i.e.* six has occured,
E_2 : die does not show six *i.e.* six has not occurred and
A : the man reports that six has occurred.
We wish to calculate the probability that six has actually occurred given that the man reports that six occurs *i.e.*, $P(E_1|A)$.

Now, $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{5}{6}$

$P(A|E_1)$ = probability that the man reports that six occurs given that six has occurred

i.e., probability that the man is telling the truth $= \frac{3}{4}$

$P(A|E_2)$ = probability that the man reports that six occurs given that six has not occurred

i.e., probability that the man does not speak truth $= \frac{1}{4}$

By Bayes' theorem,

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{6}\cdot\frac{3}{4}}{\frac{1}{6}\cdot\frac{3}{4} + \frac{5}{6}\cdot\frac{1}{4}} = \frac{3}{8}$$

28. (c): Since, $f(x) = x^n \Rightarrow f(1) = 1$
$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$

$$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$$

$$\dots \qquad \dots \; \dots \; \dots \qquad \dots$$

$$\dots \qquad \dots \; \dots \; \dots \qquad \dots$$

$$f^{n}(x) = n(n-1)(n-2) \dots 2 \cdot 1$$

$$\Rightarrow f^{n}(1) = n(n-1)(n-2) \dots 2 \cdot 1$$

We have,

$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \dots + \frac{(-1)^n f^n(1)}{n!}$$

$$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \dots + \frac{(-1)^n n(n-1)(n-2) \dots 2 \cdot 1}{n!}$$

$$= 1 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n \, {}^nC_n = (1-1)^n = 0$$

29. (c) : We know that $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$

$$\Rightarrow (25)^2 + |\vec{a} \cdot \vec{b}|^2 = (5)^2 \times (13)^2$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \sqrt{3600} = \pm 60$$

30. (b): Given $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{1}{\sqrt{1+x^2}}$

It is a linear differential equation with I.F. = $e^{\int \frac{1}{x} dx} = x$

∴ Solution is, $y \cdot x = \displaystyle\int \frac{x}{\sqrt{1+x^2}} dx = \frac{1}{2} \int \frac{2x}{\sqrt{1+x^2}} dx$

$$\Rightarrow y \cdot x = \sqrt{1+x^2} + c \quad \Rightarrow y = \frac{\sqrt{1+x^2}}{x} + \frac{c}{x}$$

31. (b): We have, $(AA^T)^T = (A^T)^T A^T = AA^T$

∴ AA^T is a symmetric matrix.

32. (b): The equation of plane is $\begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$

$$\Rightarrow 2x + 3y - 3z = 5$$

Distance of origin (0, 0, 0) from the above plane is

$$\left| \frac{-5}{\sqrt{2^2 + 3^2 + 3^2}} \right| = \frac{5}{\sqrt{22}}.$$

33. (b): $\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z$

$$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin(\pi - \sin^{-1} z) = z$$

$$\Rightarrow x\sqrt{1-y^2} = z - y\sqrt{1-x^2}$$

Squaring both sides, we get

$$x^2(1-y^2) = z^2 + y^2(1-x^2) - 2zy\sqrt{1-x^2}$$

$$\Rightarrow x^2 - y^2 - z^2 = -2zy\sqrt{1-x^2}$$

Again squaring both sides, we get

$$x^4 + y^4 + z^4 - 2x^2y^2 + 2y^2z^2 - 2x^2z^2 = 4z^2y^2 - 4x^2y^2z^2$$

$$\Rightarrow x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2x^2y^2 + 2y^2z^2 + 2z^2x^2$$

$$= 2(x^2y^2 + y^2z^2 + z^2x^2)$$

34. (a) : Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

$$= \int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + (1 - \sin^2 x)^2} dx$$

Let $\sin^2 x = t \Rightarrow 2\sin x \cos x \, dx = dt \Rightarrow \sin 2x \, dx = dt$

Also, $x = 0 \Rightarrow t = 0$ and $x = \dfrac{\pi}{2} \Rightarrow t = 1$

$$\therefore \quad I = \int_0^1 \frac{dt}{t^2 + (1-t)^2} = \frac{1}{2} \int_0^1 \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

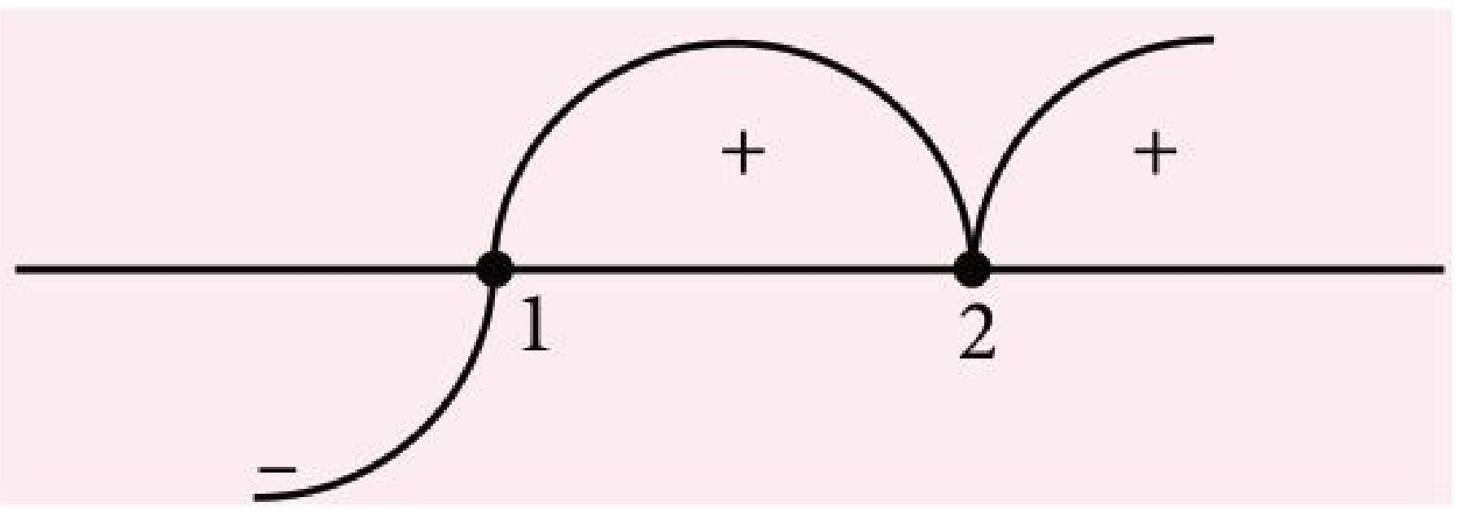
$$= \left[\tan^{-1}(2t-1)\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{2}$$

35. (b): Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\Rightarrow \text{adj}\,(\text{adj}\,A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$$

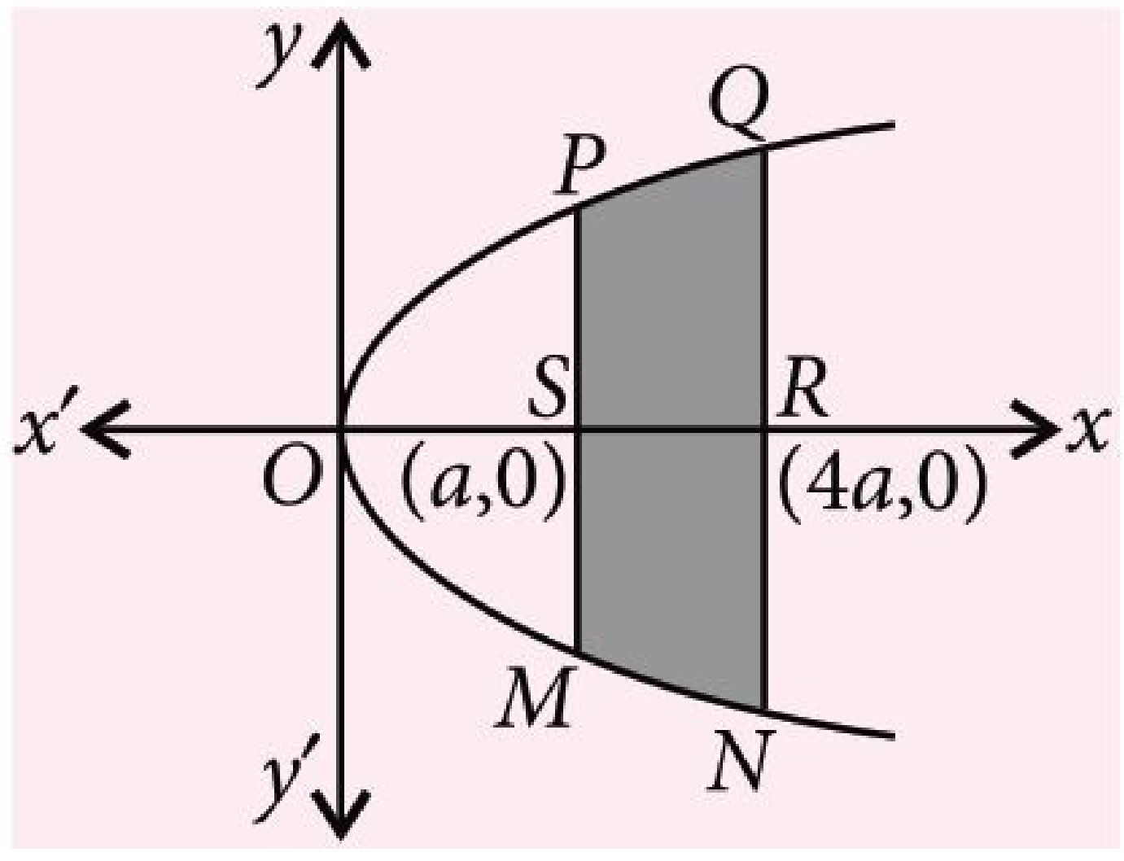
and adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow |\text{adj}\, A| = ad - bc = |A|.$

36. (c) : It is clear from figure that $f'(x)$ has no sign change at $x = 2$. Hence, $f(x)$ is neither maximum nor minimum at $x = 2$.



37. (c) : Statement-I : Required area

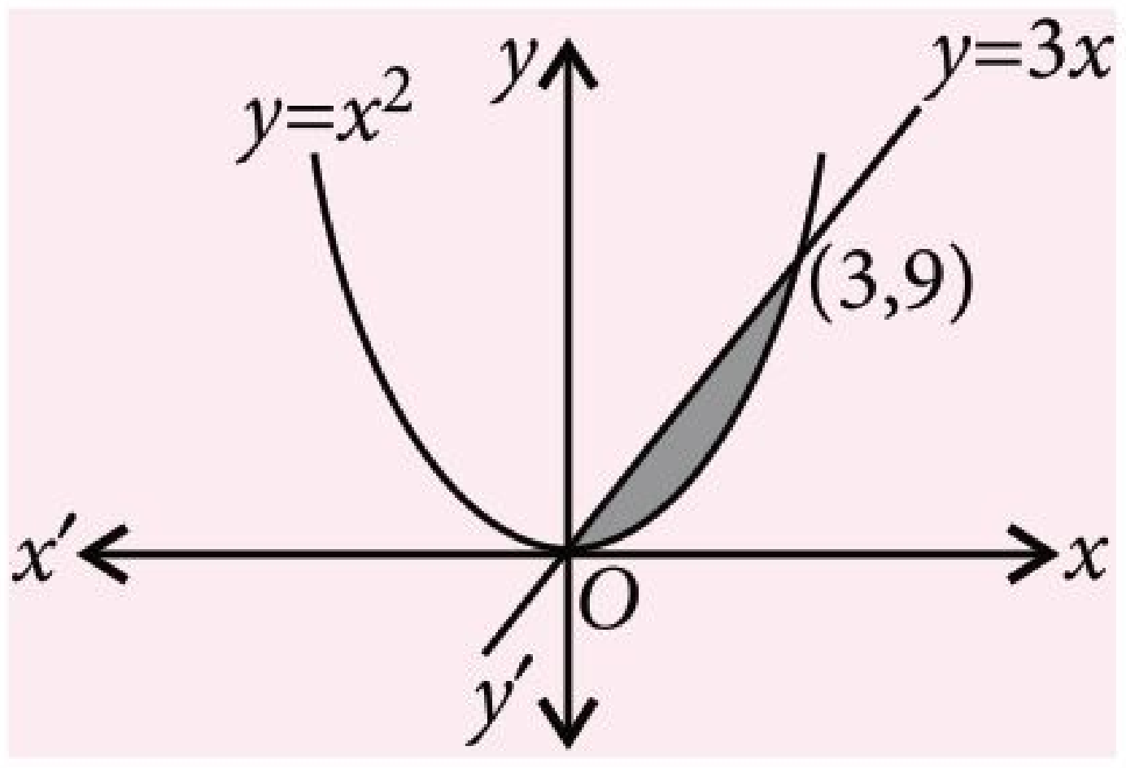
= 2 × area of region $PSRQP$



$$=2\int_a^{4a}\sqrt{4ax}\;dx=4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a}$$

$$=\frac{8}{3}\sqrt{a}\left(8a^{3/2}-a^{3/2}\right)=\frac{56a^2}{3}\text{ sq. units}$$

Statement-II : The intersection points of given curves are (0, 0) and (3, 9).



∴ Required area = $\int_0^3 (3x-x^2)\;dx$

$$=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3=\frac{27}{6}=4.5\text{ sq. units}$$

38. (c) : Here,

$\vec{N}=(2-3)\hat{i}+(-1-4)\hat{j}+(5+1)\hat{k}=-\hat{i}-5\hat{j}+6\hat{k}$ ...(i)

Now, equation of the plane passing through (2, –3, 1) perpendicular to (i) is

$-1(x-2)-5(y+3)+6(z-1)=0$

$\Rightarrow\ -x+2-5y-15+6z-6=0\Rightarrow x+5y-6z+19=0$, which is the required equation.

and length of perpendicular from (7, 14, 5) to $2x+4y-z-2=0$ is $\left|\frac{2(7)+4(14)-(5)-2}{\sqrt{2^2+4^2+(-1)^2}}\right|=3\sqrt{21}$ units.

39. (b): (P) We have, $y=\log\left(\frac{1-x^2}{1+x^2}\right)$

$$\Rightarrow\ \frac{dy}{dx}=\frac{1+x^2}{1-x^2}\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

$$=\frac{1+x^2}{1-x^2}\cdot\frac{(1+x^2)(-2x)-(2x)(1-x^2)}{(1+x^2)^2}=\frac{-4x}{1-x^4}$$

(Q) We have, $\frac{d}{dx}\left[\log(x+\sqrt{x^2+1})\right]$

$$=\frac{1}{x+\sqrt{x^2+1}}\cdot\frac{d}{dx}\left(x+\sqrt{x^2+1}\right)$$

$$=\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1+\frac{1\cdot 2x}{2\sqrt{x^2+1}}\right)$$

$$=\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}\left(x+\sqrt{x^2+1}\right)}=\frac{1}{\sqrt{x^2+1}}$$

(R) We have, $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{e^{2x}-1}{e^{2x}+1}$

$$\Rightarrow\ \frac{dy}{dx}=\frac{(e^{2x}+1)(2e^{2x})-(e^{2x}-1)(2e^{2x})}{(e^{2x}+1)^2}=\frac{4e^{2x}}{(e^{2x}+1)^2}$$

(S) Putting $x=\tan\theta$ in given equation, we get

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$=\tan^{-1}(\tan 2\theta)=2\tan^{-1}x$

and $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)=\cos^{-1}(\cos 2\theta)$

$=2\tan^{-1}x$

$\therefore\ \ y=2\tan^{-1}x-2\tan^{-1}x=0\Rightarrow\ \frac{dy}{dx}=0$

40. (c) : (P) $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)$

$$=\cos^{-1}\left\{\frac{4}{5}\times\frac{12}{13}-\sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right\}$$

$$=\cos^{-1}\left\{\frac{48}{65}-\sqrt{1-\frac{16}{25}}\sqrt{1-\frac{144}{169}}\right\}=\cos^{-1}\left\{\frac{48}{65}-\frac{3}{5}\times\frac{5}{13}\right\}$$

$$=\cos^{-1}\left\{\frac{48}{65}-\frac{15}{65}\right\}=\cos^{-1}\frac{33}{65}$$

(Q) Let $\cos\alpha=\frac{3}{5}\ \Rightarrow\ \sin\alpha=\frac{4}{5}$ and $\tan\alpha=\frac{4}{3}$

$\Rightarrow\alpha=\tan^{-1}\left(\frac{4}{3}\right)$

$\therefore\ \ \cos^{-1}\left[\frac{3}{5}\cos x+\frac{4}{5}\sin x\right]$

$=\cos^{-1}[\cos\alpha\cdot\cos x+\sin\alpha\cdot\sin x]=\cos^{-1}[\cos(\alpha-x)]$

$=\alpha-x=\tan^{-1}\frac{4}{3}-x$

(R) $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\times\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right)$$

$$= \cot^{-1}\left(\frac{(\sqrt{1+\sin x}+\sqrt{1-\sin x})^2}{(1+\sin x)-(1-\sin x)}\right)$$

$$= \cot^{-1}\left(\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{2\sin x}\right)$$

$$= \cot^{-1}\left(\frac{2+2\sqrt{1-\sin^2 x}}{2\sin x}\right) = \cot^{-1}\left(\frac{1+|\cos x|}{\sin x}\right)$$

$$= \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\left(\because x \in \left(0, \frac{\pi}{4}\right) \Rightarrow \cos x > 0 \Rightarrow |\cos x| = \cos x\right)$$

$$= \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}.$$

(S) $$\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right) = \tan^{-1}\left(\frac{\frac{a}{b}-\left(\frac{a-b}{a+b}\right)}{1+\frac{a}{b}\left(\frac{a-b}{a+b}\right)}\right)$$

$$= \tan^{-1}\left(\frac{a^2+ab-ab+b^2}{ab+b^2+a^2-ab}\right) = \tan^{-1}1 = \frac{\pi}{4}$$

41. (a): Clearly, $P(\mathrm{T}_2$ winning a match against $\mathrm{T}_1)$

$= P(\mathrm{T}_1 \text{ losing}) = \dfrac{1}{5}$

42. (d): Clearly, $P(\mathrm{T}_2$ drawing a match against $\mathrm{T}_1)$

$= P(\mathrm{T}_1 \text{ drawing}) = \dfrac{3}{10}$

43. (d): According to given information, we have the following possibilities for the values of X and Y.

X	4	3	2	1	0
Y	0	1	2	3	4

Now, $P(X > Y) = P(X = 4, Y = 0) + P(X = 3, Y = 1)$

$= P(\mathrm{T}_1 \text{ win})\, P(\mathrm{T}_1 \text{ win}) + P(\mathrm{T}_1 \text{ win})\, P(\text{match draw}) + P(\text{match draw})\, P(\mathrm{T}_1 \text{ win})$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{3}{10} + \frac{3}{10}\cdot\frac{1}{2} = \frac{5+3+3}{20} = \frac{11}{20}$$

44. (c): $P(X = Y) = P(X = 2, Y = 2)$

$= P(\mathrm{T}_1 \text{ win})\, P(\mathrm{T}_2 \text{ win}) + P(\mathrm{T}_2 \text{ win})\, P(\mathrm{T}_1 \text{ win}) + P(\text{match draw})\, P(\text{match draw})$

$$= \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{5}\cdot\frac{1}{2} + \frac{3}{10}\cdot\frac{3}{10} = \frac{1}{10} + \frac{1}{10} + \frac{9}{100} = \frac{29}{100}$$

45. (a): From the given information, it is clear that maximum sum of X and Y can be 4, therefore $P(X + Y = 8) = 0$

46. (b): We have, $\overrightarrow{OA} = 8\hat{i}$ and $\overrightarrow{AB} = 6\hat{j}$

$\therefore\ \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 8\hat{i} + 6\hat{j}$

47. (a): To reach school Geetika travels

= (8 + 6) km = 14 km

48. (b): Vector distance from school to Alok's house

$= 6\cos 30°\,\hat{i} + 6\sin 30°\,\hat{j}$

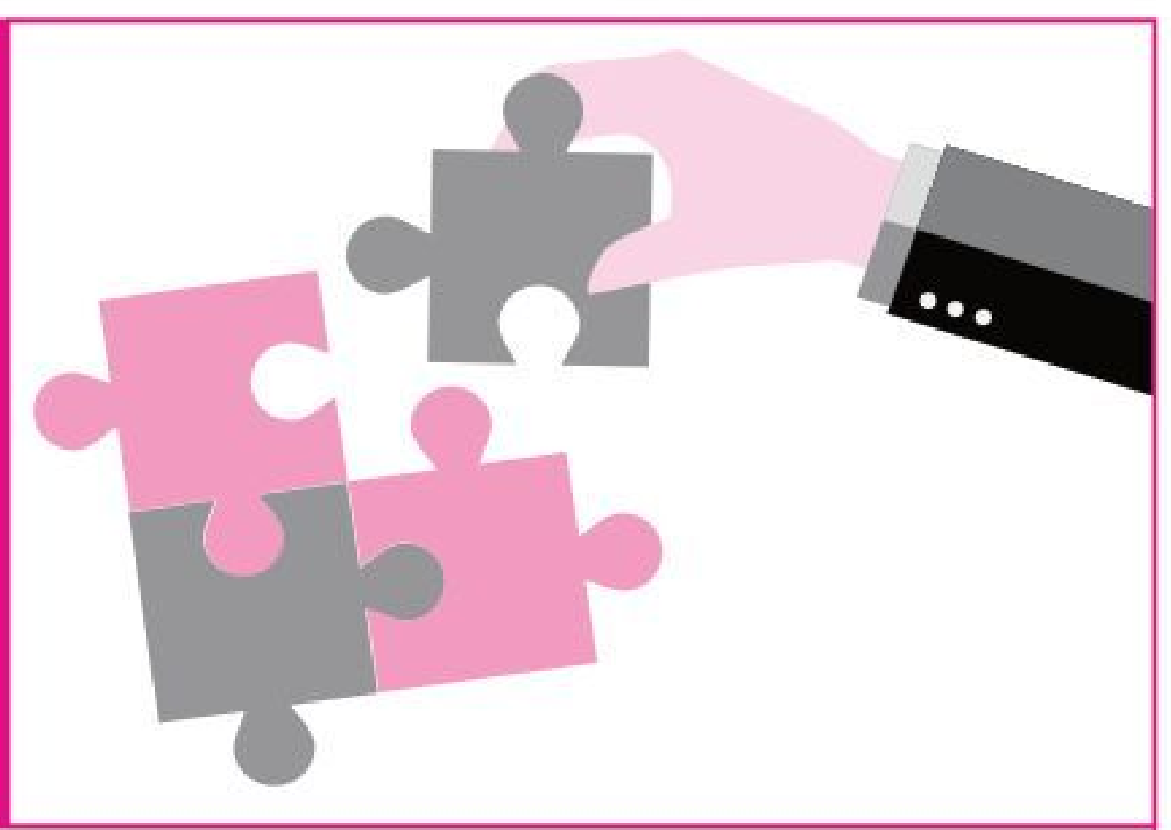
$= 6 \times \dfrac{\sqrt{3}}{2}\hat{i} + 6 \times \dfrac{1}{2}\hat{j} = 3\sqrt{3}\,\hat{i} + 3\hat{j}$

49. (a): Vector distance from Geetika's house to Alok's house $= 8\hat{i} + 6\hat{j} + 3\sqrt{3}\,\hat{i} + 3\hat{j} = (8 + 3\sqrt{3})\hat{i} + 9\hat{j}$

50. (b): Total distance travelled by Geetika from her house to Alok's house = (8 + 6 + 6) km = 20 km

◆◆

PUZZLE CORNER

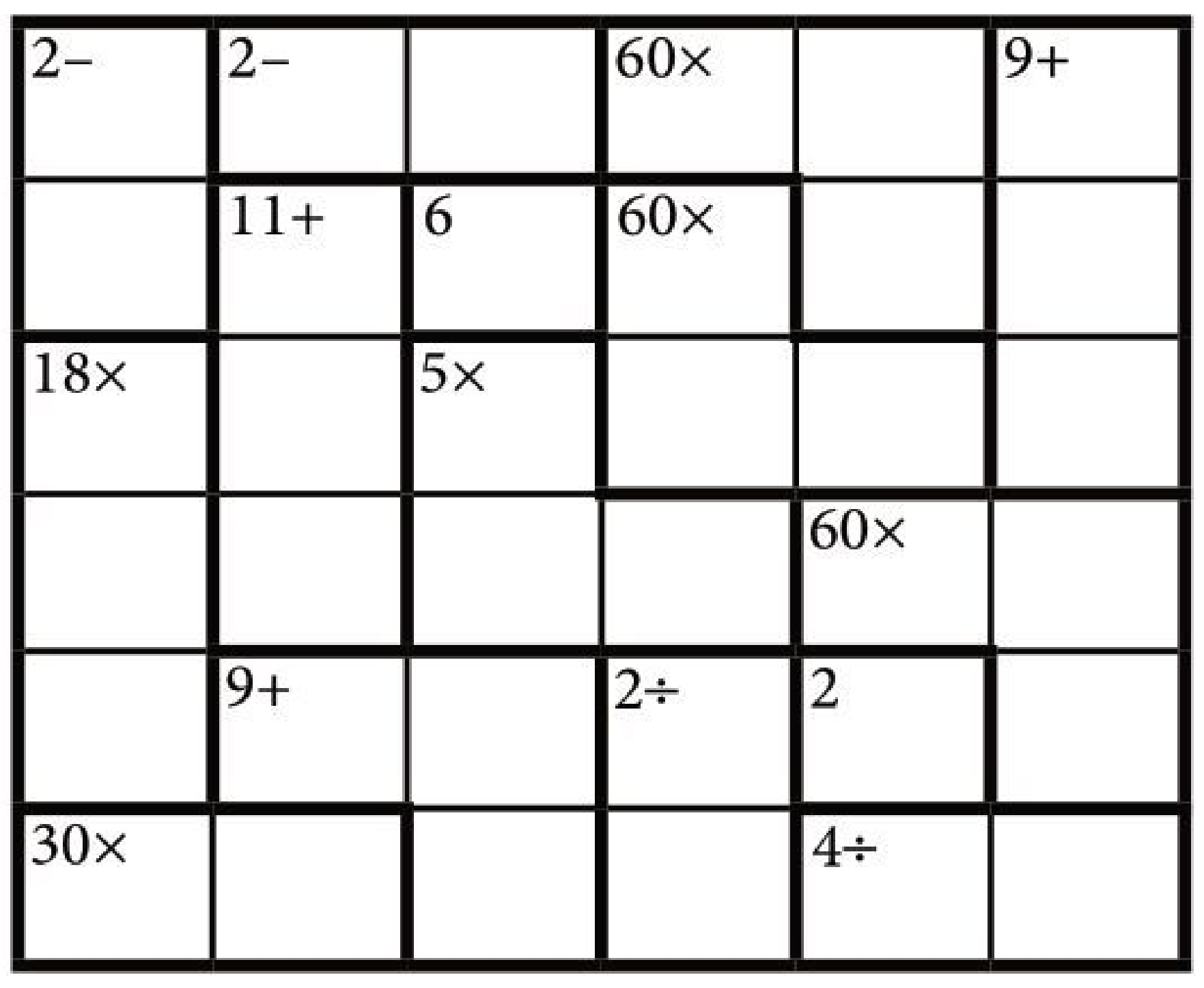


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



2−	2−		60×		9+
	11+	6	60×		
18×		5×			
				60×	
	9+		2÷	2	
30×				4÷	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

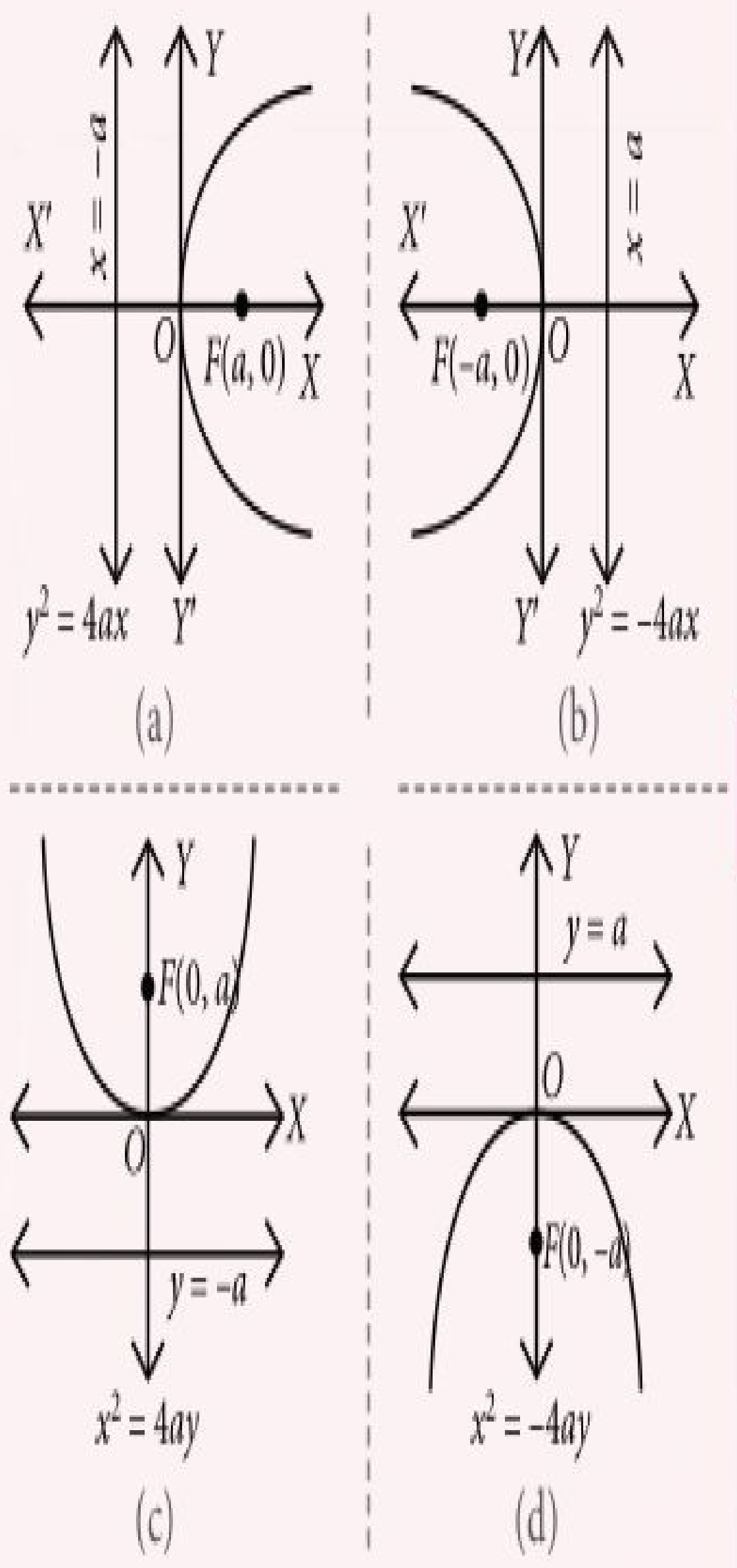
CONCEPT MAP

PARABOLA

Class XI

Definition

Parabola is locus of point which moves in a plane such that its distance from a fixed point is equal to its distance from a fixed line. The fixed point is the focus. The fixed line is the directrix.



Equation of Tangent

Curve	Point	Equation of Tangent
$y^2 = 4ax$	(x_1, y_1)	$yy_1 = 2a(x + x_1)$ (Point form)
	$(at^2, 2at)$	$ty = x + at^2$ (Parametric form)
	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ (slope is given)	$y = mx + \frac{a}{m}$ (Slope form)

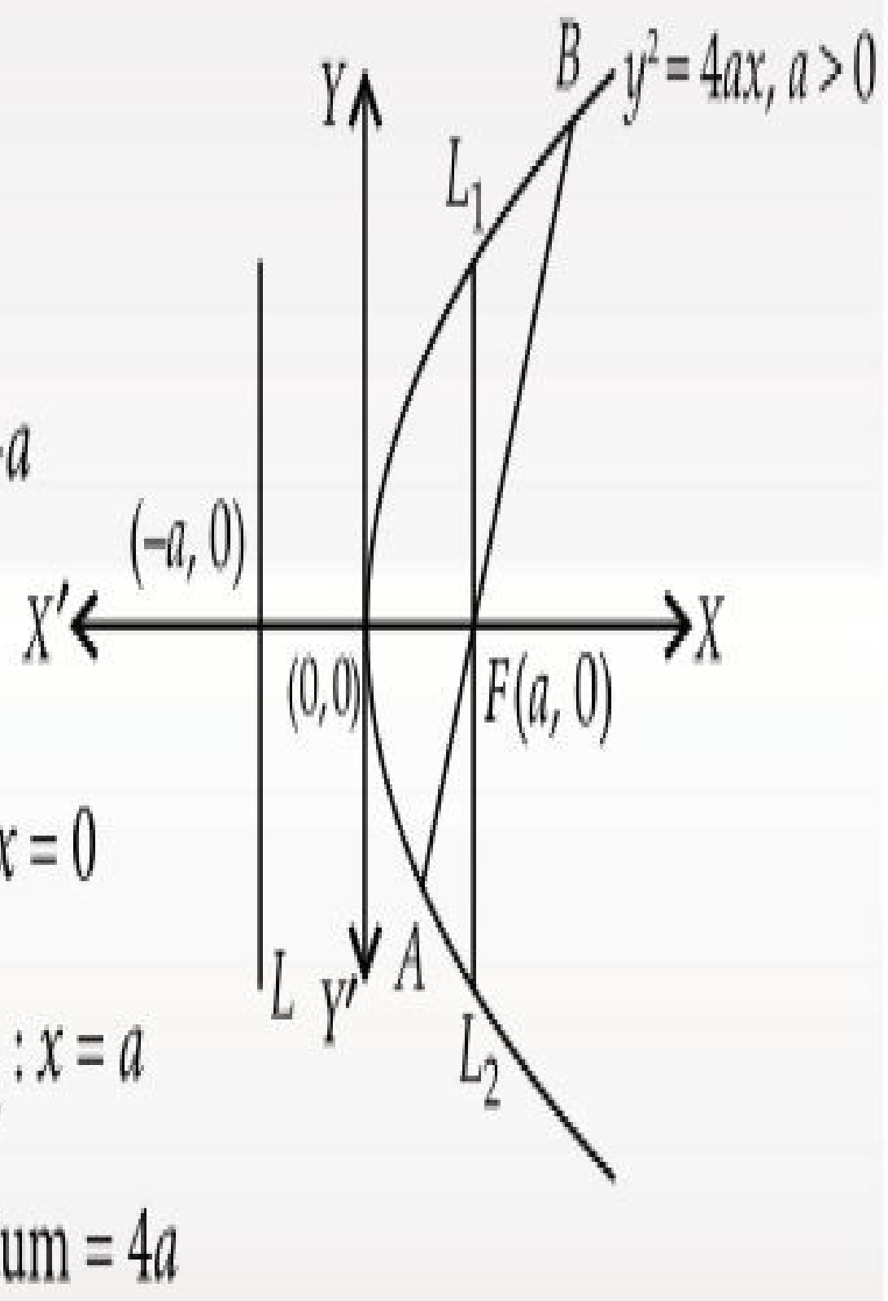
Equation of Normal

Curve	Point	Equation of Tangent
$y^2 = 4ax$	(x_1, y_1)	$y - y_1 = -\frac{y_1}{2a}(x - x_1)$ (Point form)
	$(at^2, 2at)$	$y + tx = at^3 + 2at$ (Parametric form)
	$(am^2, -2am)$	$y = mx - 2am - am^3$ (Slope form)

Standard Form

Standard form of parabola is $y^2 = 4ax$, $a > 0$

- Focus (F) : $(a, 0)$
- Vertex : $(0, 0)$
- Directrix (L) : $x = -a$
- Axis : $y = 0$
- Tangent at vertex : $x = 0$
- Latus Rectum, L_1L_2 : $x = a$
- Length of latus rectum = $4a$

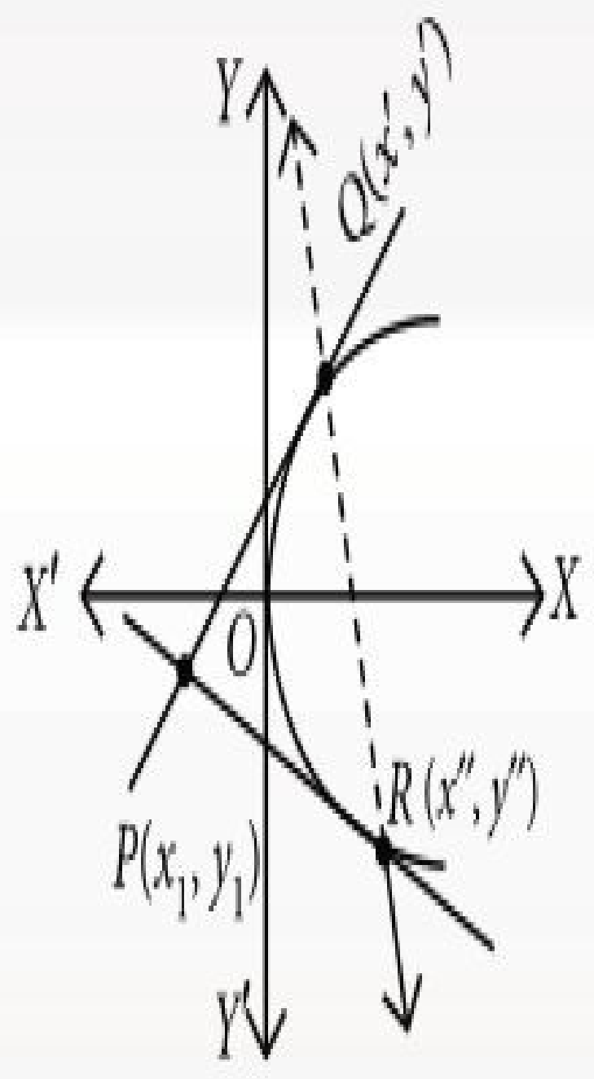


Condition of Tangency of a Line to the Parabola

Parabola	Line	Condition
$y^2 = 4ax$	$y = mx + c$	$c = \frac{a}{m}$
	$x \cos\alpha + y \sin\alpha = p$	$p = -a \tan\alpha \sin\alpha$
	$lx + my + n = 0$	$ln = am^2$

Some Important Results

- Locus of point of intersection of perpendicular tangents to the parabola is its directrix, $x = -a$.
- Equation of chord of parabola bisected at point (x_1, y_1) is given by $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$
- Equation of chord of contact (QR) of tangents to a parabola :
  - Equation of QR is given by $yy_1 = 2a(x + x_1)$
  - If $Q(at_1^2, 2at_1)$, $R(at_2^2, 2at_2)$ then equation of QR is given by $y(t_1 + t_2) = 2x + 2at_1t_2$



- The length of chord of the parabola $y^2 = 4ax$ passing through the vertex and making an angle θ with the axis is $4a \cot\theta \operatorname{cosec}\theta$.

CONCEPT MAP

TANGENTS AND NORMALS

Class XII

Equation of Tangent and Normal

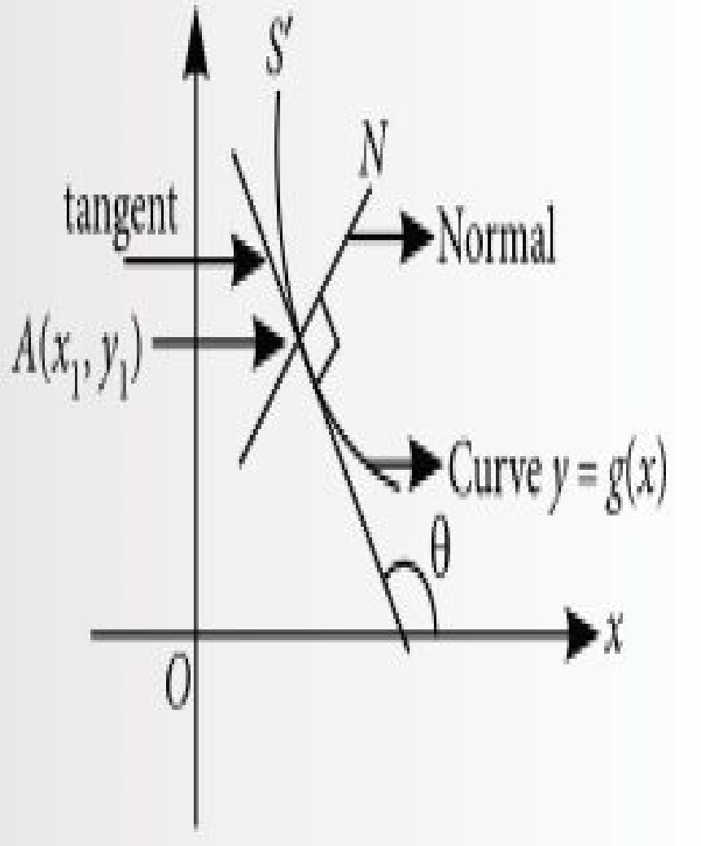
For curve $y = f(x)$

- Equation of tangent at $A(x_1, y_1)$ is given by $y - y_1 = m(x - x_1)$ or $y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} \cdot (x - x_1)$
- Equation of normal is given by $y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x - x_1)$

Note : (i) If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 0$, then tangent at $A(x_1, y_1)$ is parallel to x-axis whose equation is $y = y_1$.

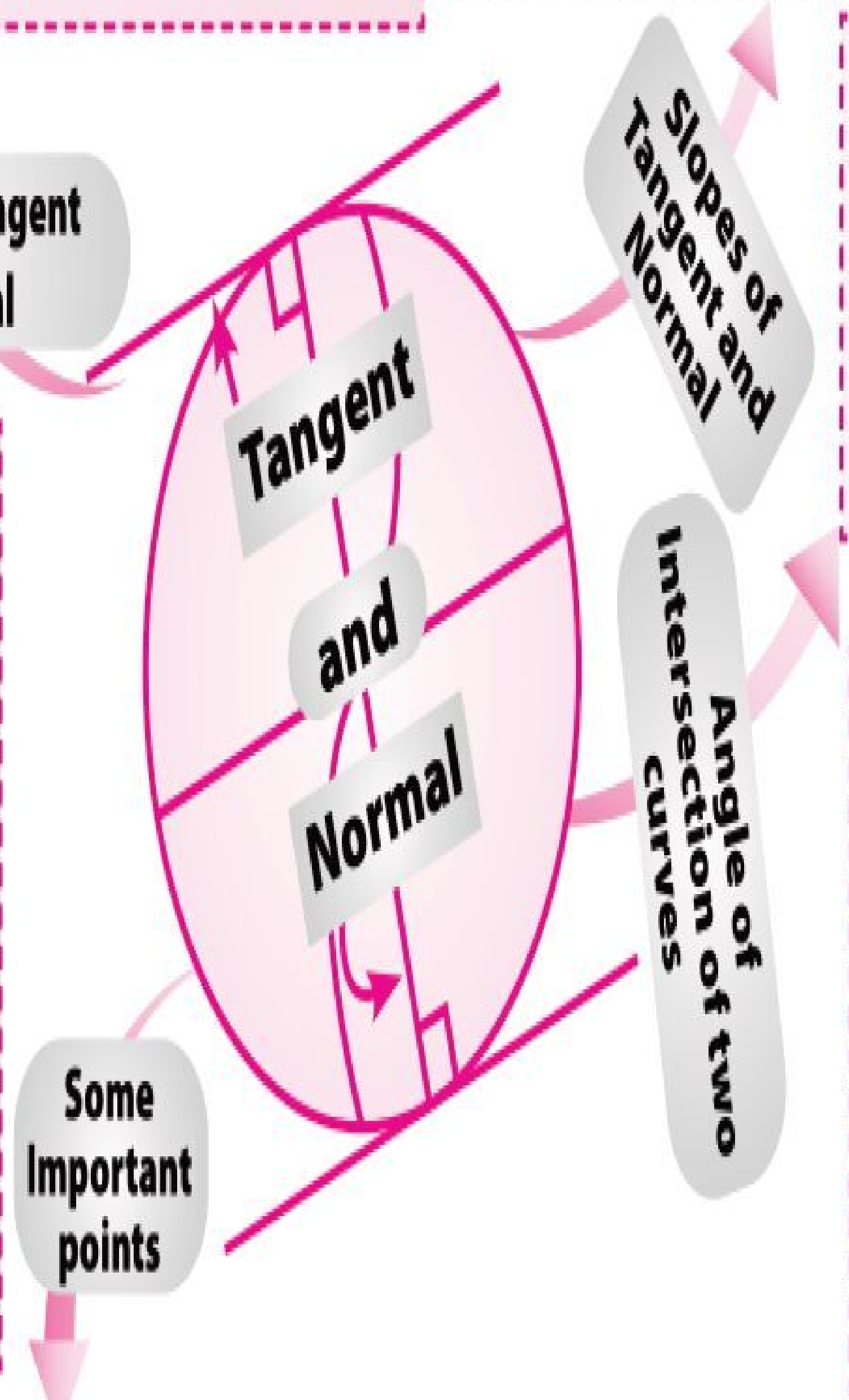
(ii) If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \infty$, then tangent at $A(x_1, y_1)$ is parallel to y-axis, whose equation is given by $x = x_1$.

- **Slope of tangent :** Let $y = g(x)$ be a continuous curve & $A(x_1, y_1)$ be any point on the curve, then $\left(\frac{dy}{dx}\right)_{A(x_1,y_1)}$ is known as the slope of the tangent denoted by tan θ to the curve $y = g(x)$ at point A, where θ is the angle made by the tangent in the positive direction of x-axis.



- **Slope of normal :** Slope of normal at $A(x_1, y_1)$ is $= -\frac{1}{\text{Slope of tangent at } A(x_1, y_1)} = -\left(\frac{dx}{dy}\right)_{A(x_1,y_1)}$

Equation of Tangent and Normal


Tangent
and
Normal
Slopes of Tangent and Normal
Angle of Intersection of two curves

Some Important points

Let $x = f(t)$, $y = g(t)$ be the equations of the curve in parametric form, then equation of tangent & normal are given by

$$y - g(t_1) = \frac{g'(t_1)}{f'(t_1)}(x - f(t_1)) \text{ and}$$

$$(y - g(t_1))\, g'(t_1) + (x - f(t_1))\, f'(t_1) = 0$$

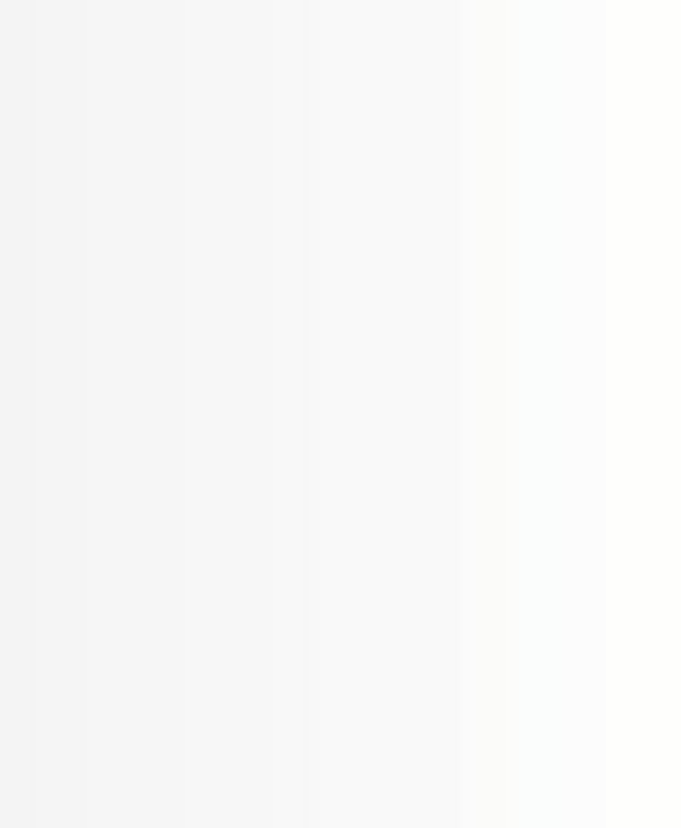
respectively.

If θ be the angle of intersection of the curves having slopes m_1 and m_2, then $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

- **Orthogonal Curves :** Two curves are said to be orthogonal if angle of their intersection is 90° *i.e.* $m_1m_2 = -1$, where m_1, m_2 are their slopes respectively.
- **Touching Curves :** Two curves touch each other when $m_1 = m_2$.

If $m = \frac{dy_1}{dx_1} = \tan\theta$, then

- **Length of tangent** $= AP = |y_1 \operatorname{cosec}\theta| = \left|y_1\sqrt{1 + \cot^2\theta}\right| = \left|\frac{y_1\sqrt{1 + \tan^2\theta}}{\tan\theta}\right| = \left|\frac{y_1\sqrt{1 + \left(\frac{dy_1}{dx_1}\right)^2}}{\frac{dy_1}{dx_1}}\right|$



- **Length of normal** $= AQ = y_1 \sec\theta = \left|y_1 \cdot \sqrt{1 + \tan^2\theta}\right| = \left|y_1\sqrt{1 + \left(\frac{dy_1}{dx_1}\right)^2}\right|$
- **Length of subtangent** $= PL = |y_1 \cot\theta| = \left|\frac{y_1}{\frac{dy_1}{dx_1}}\right|$ = Projection of tangent on the x-axis.
- **Length of subnormal** $= LQ = |y_1 \tan\theta| = \left|y_1\frac{dy_1}{dx_1}\right|$
- Intercepts of tangent at (x_1, y_1) to the curve on x-axis $= x_1 - y_1\left(\frac{dx_1}{dy_1}\right)$ and on y-axis $= y_1 - x_1\left(\frac{dy_1}{dx_1}\right)$

JEE WORK OUTS

PAPER-1

SECTION-I

INTEGER ANSWER TYPE

This section contains eight questions. The answer to each question is a single digit integer ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.

1. Let $f : R \to R$ be a differentiable function satisfying $f(x) = f(y)\, f(x - y)\ \forall\ x, y \in R$ and $f'(0) = \int_0^4 \{2x\}\, dx$, where {.} denotes the fractional part function and $f'(-3) = \alpha e^{\beta}$. Then, $|\alpha + \beta|$ is equal to ______.

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then find the maximum value of $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$

3. If $\int \frac{\ln x}{x^3} dx = -\frac{\ln x}{ax^2} - \frac{1}{bx^2} + c$, then the value of $a + b$ must be ______.

4. TP and TQ are any two tangents to a parabola and the tangent at a third point R cuts them in P' and Q' respectively, then the value of $\frac{TP'}{TP} + \frac{TQ'}{TQ}$ must be ___.

5. If α, β, γ, δ are the solutions of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$, no two of which have equal tangents, then the value of $\tan\alpha + \tan\beta + \tan\gamma + \tan\delta$ is equal to ______.

6. The number of solutions for the equation $\log_4 (2x^2 + x + 1) - \log_2 (2x - 1) = 1$, is ______.

7. Find the value of

$$4\left(\cos\frac{2\pi}{15} + \cos\frac{4\pi}{15} - \cos\frac{7\pi}{15} - \cos\frac{\pi}{15}\right).$$

8. If $(y^2 - 5y + 3)(x^2 + x + 1) < 2x$ for all $x \in R$, then number of integral values of y will be

SECTION-II

ONE OR MORE THAN ONE OPTION CORRECT TYPE

This section contains ten questions. Each question has four options (a), (b), (c) and (d). One or More than one of these four option (s) is (are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened, 0 If none of the bubbles is darkened, –2 In all other cases.

9. If a, b, c are real numbers and z is a complex number such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then $\frac{1 + iz}{1 - iz}$ equals

(a) $\frac{b - ic}{1 - ia}$ (b) $\frac{a + ib}{1 + c}$ (c) $\frac{1 - c}{a - ib}$ (d) $\frac{1 + a}{b + ic}$

10. If the quadratic polynomial $P(x) = (p - 3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be

(a) $\frac{3}{2}$ (b) 4 (c) 6 (d) 7

11. All non-zero complex numbers on the complex plane satisfying $z + \frac{1}{z} \in R$ can lie on

(a) $y = 0$ (b) $x = 0$

(c) $y = x$

(d) a unit circle with centre at the origin

12. Find the domain of $f(x) = \sin^{-1}[2 - 4x^2]$ ([.] denotes the greatest integer function)

(a) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$ (b) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

(c) $\left[-\frac{\sqrt{3}}{2}, 0\right)$ (d) $\left[-\frac{\sqrt{3}}{2}, 8\right]$

13. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\sum_{n=1}^{n} n^3$

(c) $\frac{n(n+1)(2n+1)}{6}$ (d) ${}^{n+2}C_3$

14. The line $3x + 6y = k$ intersect the curve $2x^2 + 2xy + 3y^2 = 1$ at points A and B. The circle on AB as diameter passes through the origin. The possible value(s) of k is (are)

(a) 3 (b) 4 (c) –4 (d) –3

15. Let $P(x) = x^2 + bx + c$, where b and c are integers. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then

(a) $P(x) = 0$ has imaginary roots

(b) $P(x) = 0$ has roots of opposite sign

(c) $P(1) = 4$ (d) $P(1) = 6$

16. $$\left[({}^nC_0 + {}^nC_3 + ...) - \frac{1}{2}({}^nC_1 + {}^nC_2 + {}^nC_4 + {}^nC_5 + ...)\right]^2$$
$$+\frac{3}{4}({}^nC_1 - {}^nC_2 + {}^nC_4 - {}^nC_5 + ...)^2 =$$

(a) 3 (b) 4 (c) 2 (d) 1

17. For every pair of continuous functions $f, g : [0, 1] \to R$, such that $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is (are):

(a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

(b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

(c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$

(d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

18. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is

(a) 5 (b) 20 (c) 10 (d) 30

SECTION-III

MATRIX MATCH TYPE

This section contains 2 questions. Each question contains two columns, Column I and Column II. Column I has four entries (A), (B), (C) and (D). Column II has four entries (p), (q), (r) and (s). Match the entries in Column I with the entries in Column II. One or more entries in Column I may match with one or more entries in Column II. The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown. For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I matches with entries (q), (r) and (s), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D). Marking scheme : For each entry in Column I. +2 If only the bubble(s) corresponding to all the correct match(es) is (are) darkened, 0 If none of the bubbles is darkened, –1 In all other cases.

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

19. Match the following: -

	Column I		Column II
(A)	Consider the differential equation, $y = x\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ It's degree is	(p)	1
(B)	The differential equation representing the family of curve $y^2 = 2c(x + \sqrt{c})$, where c is a parameter, is of degree	(q)	4
(C)	Consider differential equation, $1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...\infty = y,$ then its degree is	(r)	2
(D)	Consider the differential equation, $\frac{dy}{dx} = \left(1 + \left(\frac{d^2y}{dx^2}\right)^2\right)^{2/3}$ Its degree is	(s)	3

20. Consider the system of equations

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

Now, let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ and $\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

According to Crammer's rule,

$$x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$$

Match the following: -

	Column I		Column II
(A)	If $\Delta \neq 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$, then the solutions are	(p)	Unique
(B)	If $\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$, then the solutions are	(q)	Inconsistent
(C)	If $\Delta \neq 0$ and atleast one of Δ_1, Δ_2, Δ_3 is non-zero, then the solutions are	(r)	Non-Trivial
(D)	If atleast one of Δ_1, Δ_2, Δ_3 is non-zero, and $\Delta = 0$, then the solutions are	(s)	Infinitely many

PAPER-2

SECTION-I

INTEGER ANSWER TYPE

This section contains eight questions. The answer to each question is a single digit integer ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.

1. Let $f(x, y)$ be a periodic function satisfying the condition $f(x, y) = f((2x + 2y), (2y - 2x))\ \forall\ x, y \in R$. Now, define a function g by $g(x) = f(2^x, 0)$. If period of $g(x)$ is α, then value of $|\alpha - 7|$ is

2. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is

3. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k\sqrt{x^2 + y^2} = 0$, find k.

4. If $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$ for all $x \in R$, then find the value of $f(1)$.

5. Find $\lim\limits_{x \to \infty} \left| \dfrac{(3x^4 + 2x^2)\sin\frac{1}{x} + |x|^2 + 5}{|x|^3 + |x|^2 + |x| + 1} \right|$

6. If $y(t)$ is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1) + \dfrac{3}{2}$ is equal to

7. Suppose that f is differentiable for all x and that $f'(x) \leq 2$ for all x. If $f(1) = 2$ and $f(4) = 8$, then $f(2)$ has the value equal to______.

8. Two whole numbers are randomly selected and multiplied. If the probability that the units place in their product is "Even" is p and the probability that the units place in their product is "odd" is q, then $\left(\dfrac{p}{q}\right) + 1$ is equal to

SECTION-II

ONE OR MORE THAN ONE OPTION CORRECT TYPE

This section contains eight questions. Each question has four options (a), (b), (c) and (d). One or More than one of these four option (s) is (are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened, 0 If none of the bubbles is darkened, –2 In all other cases.

9. The point of intersection of the plane $\vec{r}.(3\hat{i} - 5\hat{j} + 2k) = 6$ with the straight line passing through the origin and perpendicular to the plane $2x - y - z = 4$ is

(a) $(1, -1, -1)$ (b) $(-1, -1, 2)$

(c) $(4, 2, 2)$ (d) $\left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

10. If the median through A of a ΔABC having vertices $A \equiv (2, 3, 5)$, $B \equiv (-1, 3, 2)$ and $C \equiv (\lambda, 5, \mu)$ is equally inclined to the axes, then

(a) $\lambda = 7$ (b) $\mu = 10$

(c) $\lambda = 10$ (d) $\mu = 7$

11. The integral of $\dfrac{1}{\sin^2 x + \tan^2 x}$ must be

(a) $-\dfrac{1}{2}\left[\tan x + \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$

(b) $-\dfrac{1}{2}\left[\cot x + \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$

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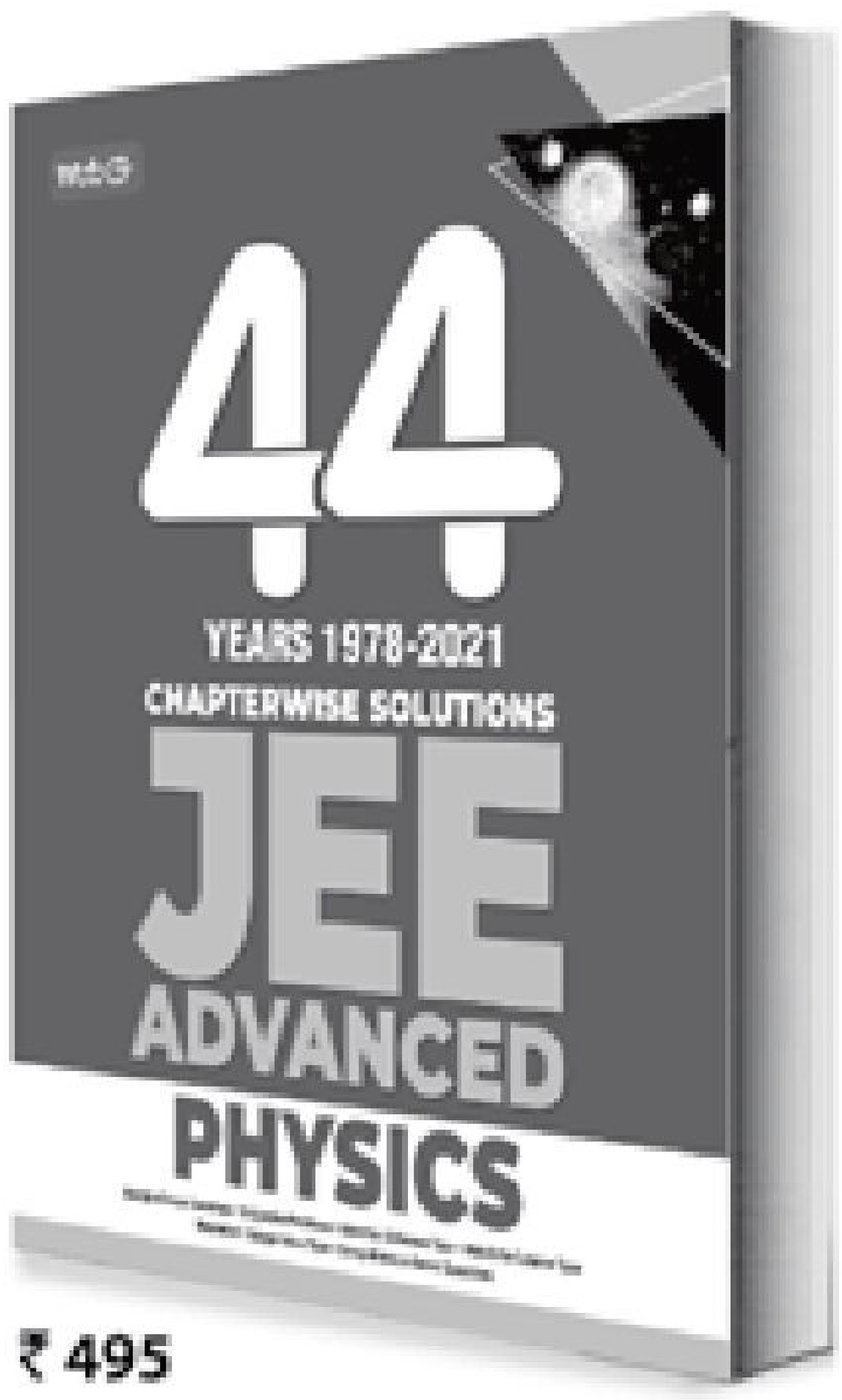
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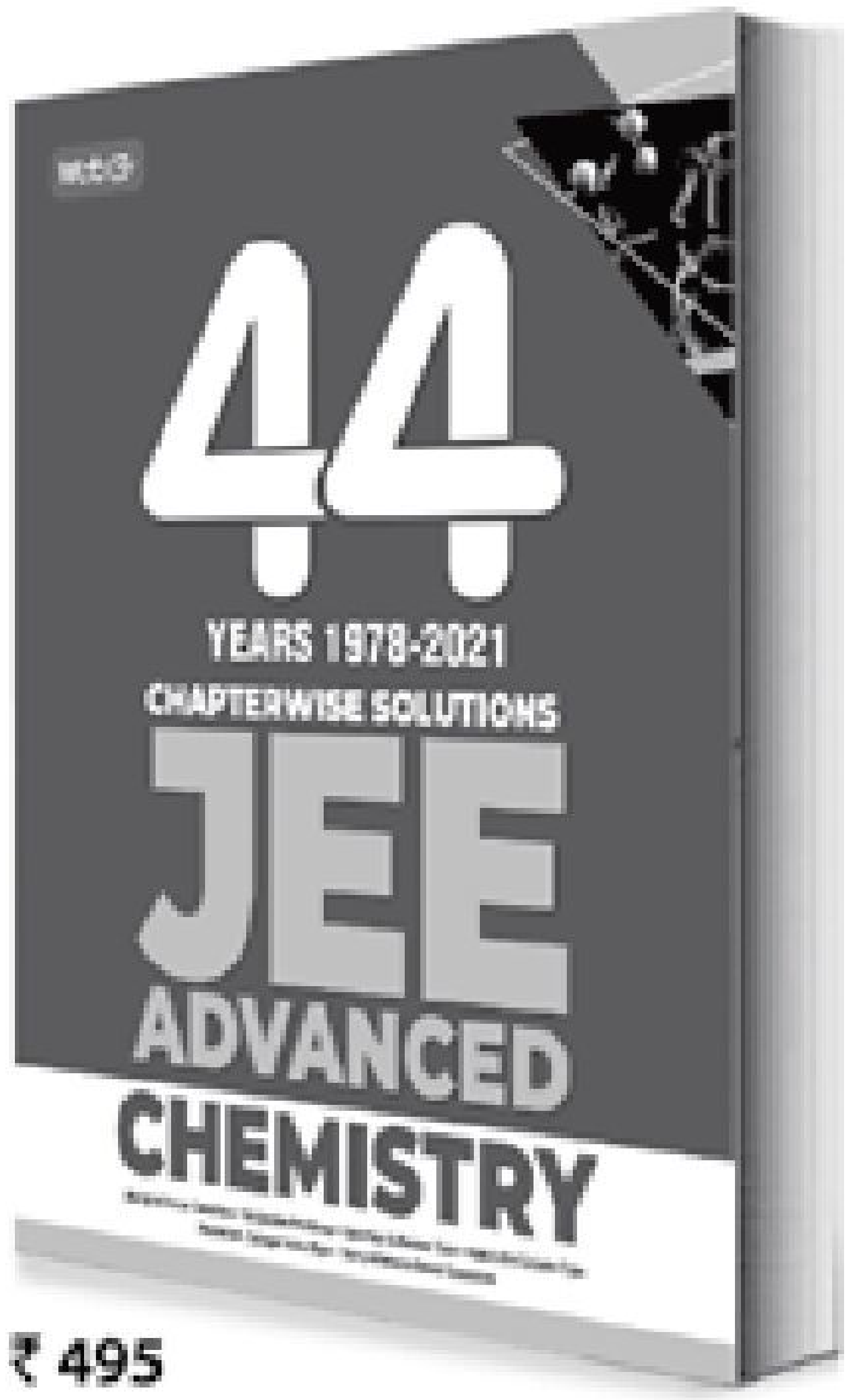
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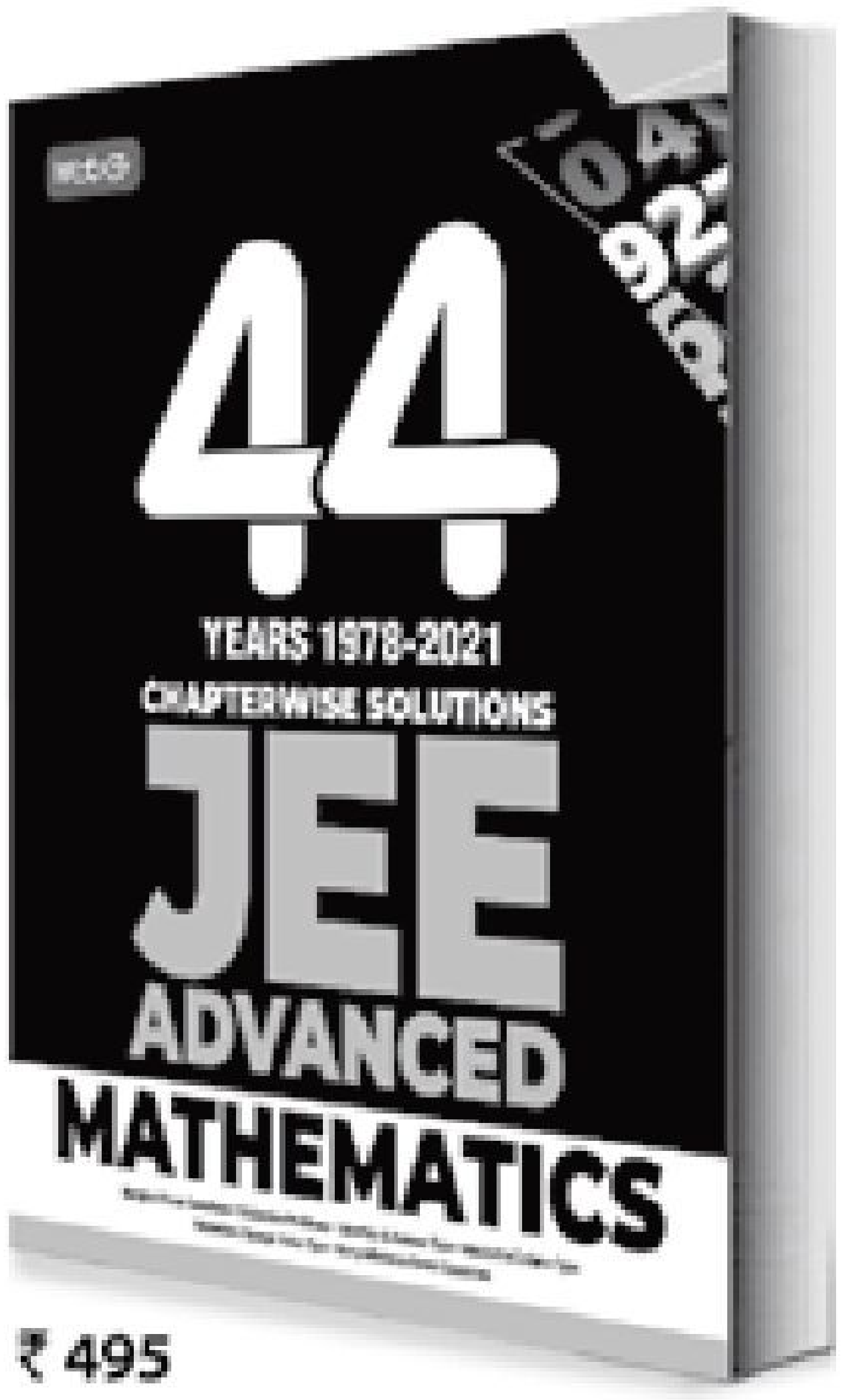
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(c) $-\frac{1}{2}\left[\cot x-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

(d) $-\frac{1}{2}\left[\cot x-\frac{1}{\sqrt{2}}\cot^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

12. $I=\int\sqrt{\frac{\cos^3 x}{\sin^{11} x}}\,dx$ is equal to

(a) $-\frac{2}{5}\cot^{3/2}x-\frac{2}{9}\cot^{9/2}x+c$

(b) $-\frac{2}{5}\cot^{5/2}x-\frac{2}{9}\cot^{9/2}x+c$

(c) $-\frac{2}{5}\cot^{5/2}x-\frac{2}{9}\cot^{3/2}x+c$

(d) None of these

13. The line $y = x + 5$ touches

(a) the parabola $y^2 = 20x$

(b) the ellipse $9x^2 + 16y^2 = 144$

(c) the hyperbola $\frac{x^2}{29}-\frac{y^2}{4}=1$

(d) the circle $x^2 + y^2 = 25$

14. How many numbers between 5,000 and 10,000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number ?

(a) 1680 (b) $5 \times {}^8P_8$

(c) $30 \times {}^8C_3$ (d) $5! \times {}^8C_3$

15. Triplets (p, q, r) is chosen from a set {1, 2, 3, 4, 5, 6, 7, 8, 9} such that $x < y \le z$. There could be …… such triplets.

(a) 143 (b) 200 (c) 180 (d) 120

16. If in triangle ABC, CD is the angle bisector of the angle ACB, then CD is equal to

(a) $\frac{a+b}{2ab}\cos\frac{C}{2}$ (b) $\frac{a+b}{ab}\cos\frac{C}{2}$

(c) $\frac{2ab}{a+b}\cos\frac{C}{2}$ (d) $\frac{b\sin A}{\sin\left(B+\frac{C}{2}\right)}$

SECTION-III

COMPREHENSION TYPE

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each of these questions has 4 choices (a), (b), (c) and (d), out of which only one is correct. Correct answer 3 marks and wrong answer –1.

Paragraph for Question. No. 17 and 18

Consider a hyperbola whose centre is at origin. A line $x + y = 2$ touches this hyperbola at $P(1, 1)$ and intersect the asymptotes at A and B such that $AB = 6\sqrt{2}$ units.

17. Equation of asymptotes are

(a) $2x^2 + 5xy + 2y^2 = 0$ (b) $2x^2 - 5xy - 2y^2 = 0$

(c) $2x^2 - 5xy + 2y^2 = 0$ (d) $2x^2 + 5xy - 2y^2 = 0$

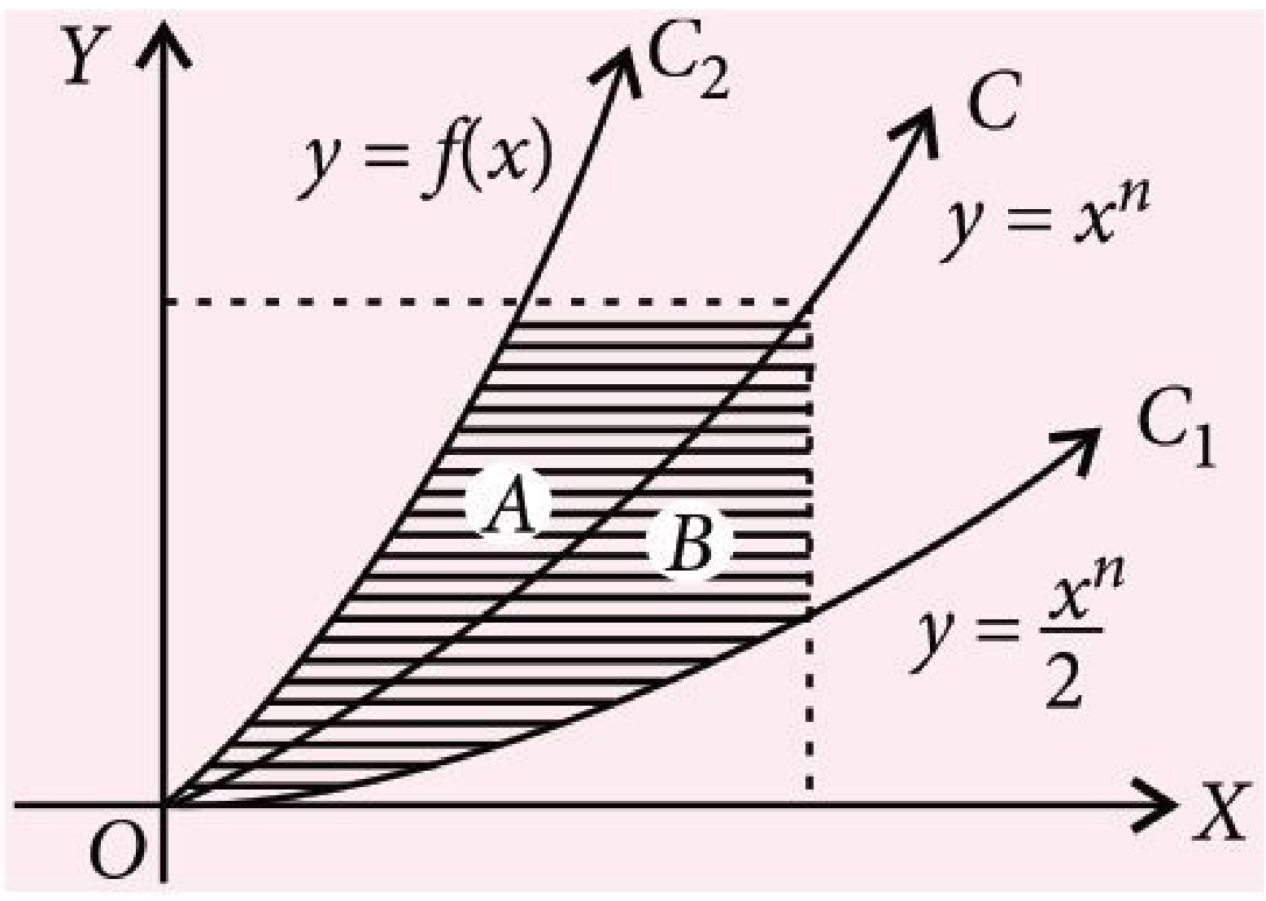
18. Equation of tangent to the hyperbola at $\left(-1,\frac{7}{2}\right)$ is

(a) $3x + 2y = 2$ (b) $3x + 4y = 11$

(c) $3x + 2y = 4$ (d) $3x + 2y = 7$

Paragraph for Question. No. 19 and 20

Let C_1 and C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" between C_1 and C_2, if for each point P of C, the two shaded regions A and B shown in the figure have equal areas. Let the equation of the curve C_1 be $y=\frac{x^n}{2}$ and that of C be $y = x^n$.



19. Area of the total shaded region $(A + B)$, if x– coordinate of P is x and $n = 2$ is

(a) $\frac{x^2}{2}$ (b) $\frac{x^3}{2}$ (c) $\frac{x^3}{3}$ (d) $\frac{x^3}{6}$

20. Equation of the curve C_2 for $n = 2$ is $y =$

(a) $\frac{3x^3}{4}$ (b) $\frac{4x^2}{3}$ (c) $\frac{9x^2}{16}$ (d) $\frac{16x^2}{9}$

ANSWER KEY

Paper-1

1.	(4)	**2.**	(9)	**3.**	(6)	**4.**	(1)
5.	(0)	**6.**	(1)	**7.**	(2)	**8.**	(2)
9.	(b, c)	**10.**	(c)	**11.**	(a, d)	**12.**	(a)
13.	(a, d)	**14.**	(a, d)	**15.**	(a, c)	**16.**	(d)
17.	(a, d)	**18.**	(c)				

19. (A) → (r), (B) → (s), (C) → (p), (D) → (q)

20. (A) → (p), (B) → (r, s), (C) → (p, r), (D) → (q)

Paper-2

1.	(5)	**2.**	(6)	**3.**	(1)	**4.**	(4)
5.	(3)	**6.**	(1)	**7.**	(4)	**8.**	(4)
9.	(d)	**10.**	(a, b)	**11.**	(b, d)	**12.**	(b)
13.	(a, b, c)	**14.**	(a, c)	**15.**	(d)	**16.**	(c, d)
17.	(a)	**18.**	(c)	**19.**	(c)	**20.**	(d)

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Challenging PROBLEMS

ON

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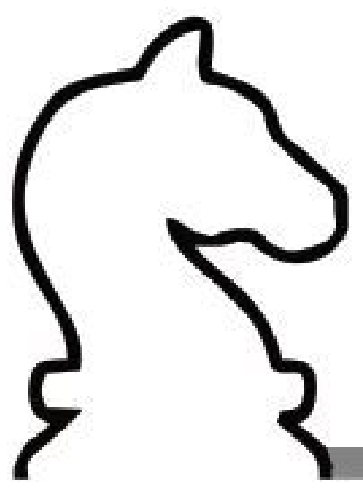

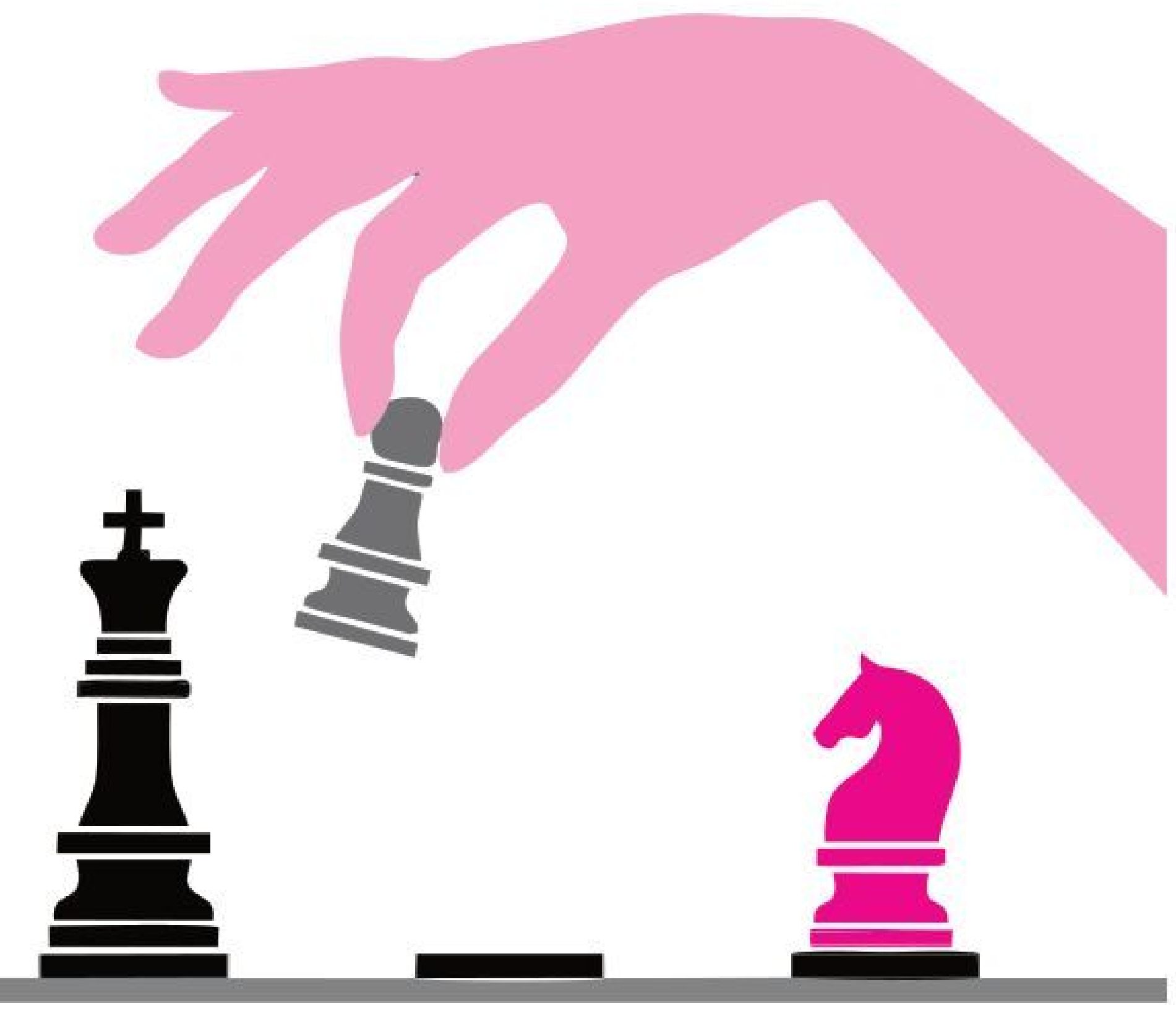

INTEGER ANSWER TYPE

1. If the solution of the differential equation $e^{3x}(p-1)+p^3e^{2y}=0$ is in the form $e^y=ce^x+c^k$ then $k=$ _____(Where $p=\dfrac{dy}{dx}$ and 'c' is arbitrary constant)

2. If the solution of the differential equation $y^2(y-xp)=x^4p^2$ is in the form $\dfrac{1}{y}=\dfrac{c}{x}+c^k$, then $k=$ __(Where $p=\dfrac{dy}{dx}$ and 'c' is arbitrary constant)

3. The solution of $x^3\dfrac{dy}{dx}+4x^2\tan y=e^x\sec y$ satisfying $y(1)=0$ is $\sin y=e^x(x-1)x^{-k}$ then $k=$ ____.

4. Differential equation, having $y=(\sin^{-1}x)^2+A(\cos^{-1}x)+B$, where A and B are arbitary constants is $(p-x^2)\dfrac{d^2y}{dx^2}-\dfrac{xdy}{dx}=q$ then $p+q=$ _____.

5. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from normal at any point 'P' of the curve is equal to the distance of P from the x-axis is a circle with radius

6. The real value of m for which the substitution $y=u^m$ will transform the differential equation $2x^4y\dfrac{dy}{dx}+y^4=4x^6$ into a homogeneous equation is $\dfrac{\lambda}{2}$. Then the value of λ is ______.

7. Let $y=f(x)$ be a curve passing through(e, e^e) which satisfy the differential equation $(2ny+xy\log_e x)\,dx-x\log_e x\,dy=0$, $x>0$, $y>0$. If $g(x)=\lim\limits_{n\to\infty}(0.020)f(x)$

Then $\displaystyle\int_{1/e}^{e}g(x)dx$ equal to ______.

MULTIPLE CORRECT ANSWER TYPE

8. A normal is drawn at a point $P(x, y)$ of a curve. It meet the x-axis at Q. If PQ is of constant length k. Then

(a) The differential equation describing such curves is $y\dfrac{dy}{dx}=\pm\sqrt{k^2-y^2}$

(b) The curve is passing through $(0, k)$

(c) The curve is passing through $(k, 0)$

(d) The equation of the curve represents circle with centre as origin

9. The orthogonal trajectories of the system of curves $\left(\dfrac{dy}{dx}\right)^2=\dfrac{a}{x}$ are

(a) $9a(y+c)^2=4x^3$ (b) $y+c=\dfrac{-2}{3\sqrt{a}}x^{3/2}$

(c) $y+c=\dfrac{2}{3\sqrt{a}}x^{3/2}$ (d) None of these

10. Solution of the differential equation

$$\frac{x+y\dfrac{dy}{dx}}{y-x\dfrac{dy}{dx}}=\frac{x\sin^2(x^2+y^2)}{y^3}$$

(a) $-\cot(x^2+y^2)=\left(\dfrac{x}{y}\right)^2+c$

(b) $\dfrac{y^2}{x^2+y^2c}=-\tan(x^2+y^2)$

(c) $-\cot(x^2+y^2)=\left(\dfrac{y}{x}\right)^2+c$

(d) $\dfrac{y^2+x^2c}{x^2}=-\tan^2(x^2+y^2)$

11. Let C be a curve such that the normal at any point P on it meets x-axis and y-axis at A and B respectively. If $BP : PA=1 : 2$ (internally) and the curve passes

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through the point (0, 4) then which of the following alternative(s) is/are correct?

(a) The curves passes through $(\sqrt{10}, -6)$

(b) The equation of tangent at $(4, 4\sqrt{3})$ is $2x+\sqrt{3}y=20$

(c) The differential equation for the curve is $yy' + 2x = 0$

(d) The curve represent a hyperbola

12. The general solution of the differential equation $\frac{dy}{dx}+\frac{y\ln y}{x}=\frac{y}{x^2}(\ln y)^2$ is, (c being the constant of integration)

(a) $2x = (1 - cx^2)\ln y$ (b) $x = (1 + 2cx^2)\ln y$

(c) $2x = (1 + 2cx^2)\ln y$ (d) $x = (1 + cx^2)\ln y$

13. If the solution of $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=x,\ y(0)=y(1)=1$ is given by $y^2 = f(x)$ then

(a) $f(x)$ is monotonically increasing $\forall\ x \in (1, \infty)$

(b) $f(x) = 0$ has only one root

(c) $f(x)$ is neither even nor odd

(d) $f(x)$ has 3 real roots

14. The coordinates of a point $P(x, y)$ are functions of time t and satisfy the relations $\frac{dx}{dt}+\frac{dy}{dt}=t$ and $\frac{dx}{dt}-2\frac{dy}{dt}=t^2$ at any instant of time t. The locus of point $P(x, y)$ is a curve given by (assume $x(0) = y(0) = 0$)

(a) $(x + y)^3 = (x - 2y)^2$

(b) $x=\frac{t^2}{3}+\frac{t^3}{9},\ y=\frac{t^2}{6}-\frac{t^3}{9}$

(c) $9(x + y)^3 = 8(x - 2y)^2$

(d) $8(x + y)^3 = 9(x - 2y)^2$

15. Solution of the differential equation :

$(3 \tan x + 4 \cot y - 7) \sin^2 y\, dx -$
$(4 \tan x + 7 \cot y - 5) \cos^2 x dy = 0$ is

(a) $\frac{3}{2}\cot^2 x-7\cot x+\frac{7}{2}\tan^2 +$
$y - 5\tan y + 4\cot x\cdot\tan y = c$

(b) $\frac{3}{2}\tan^2 x-7\tan x+\frac{7}{2}\cot^2 y-5\cot y+4\tan x\cdot\cot y=\mathrm{c}$

(c) $3 \tan^2 y - 14\cot x\cdot\tan^2 y + 7 \cot^2 x - 10 \tan y \cot^2 x$
$+ 8 \cot x\cdot\tan y + 2c \cot^2 x \tan^2 y = 0.$

(d) $3 \cot^2 y - 14\cot x\cdot\cot^2 y + 7 \cot^2 x + 10 \cot y \tan^2 x$
$+ 8 \tan x\cdot\cot y = 0.$

SOLUTIONS

1. (3) : (i) put $e^x = X$ and $e^y = Y$

So that $e^x dx = dX$ and $e^y dy = dY$

$\Rightarrow \frac{e^y}{e^x}\cdot\frac{dy}{dx}=\frac{dY}{dX}$ or $\frac{Y}{X}p=P$ (say) or $p=\frac{X}{Y}\cdot P$

$\therefore$ The given equation becomes

$$X^3\left(\frac{X}{Y}P-1\right)+\frac{X^3}{Y^3}.Y^2P^3=0$$

or $XP - Y + P^3 = 0$ or $Y = PX + P^3$

Which is of Clairaut's form.

$\therefore$ The solution is $Y = cX + c^3$ or $e^y = ce^x + c^3$

2. (2) : (i) Put $x=\frac{1}{X}$ and $y=\frac{1}{Y}$

$dx=-\frac{1}{X^2}dX$ and $dy=-\frac{1}{Y^2}dY$

$\Rightarrow p=\frac{dy}{dx}=\frac{X^2}{Y^2}\frac{dY}{dX}=\frac{X^2}{Y^2}P$ (say)

The given equation becomes

$$\frac{1}{Y^2}\left(\frac{1}{Y}-\frac{1}{X}\cdot\frac{X^2}{Y^2}P\right)=\frac{1}{X^4}\cdot\frac{X^4}{Y^4}P^2$$

$\Rightarrow$ $Y - XP = P^2$ or $Y = PX + P^2$

Which is the Claraut's form

$\therefore$ The solution is $Y = cX + c^2$ or $\frac{1}{y}=\frac{c}{x}+c^2$

3. (4) : Put $\sin y=t \Rightarrow \cos y\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore \frac{dt}{dx}+\frac{4}{x}t=\frac{e^x}{x^3}$

$I.F.=e^{\int 4/x dx}=e^{4l\mathrm{n}x}=x^4$; $\frac{d(t\cdot x^4)}{dx}=xe^x$

$\Rightarrow$ $\sin y = e^x(x - 1)x^{-4}$

4. (3) : $\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}}$

Again differentiating, we get

$\Rightarrow$ $(1 - x^2)y'' - xy' = 2$

5. (1) : Equation of normal at the point $p(x, y)$ is

$$Y-y=-\frac{dx}{dy}(X-x)$$

Let $m=\frac{dy}{dx}$ $\Rightarrow X+mY-(x+my)=0$

Distance of perpendicular from the origin to line (i) is $\frac{|x+my|}{\sqrt{1+m^2}}=|y|$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

This is homogeneous equation

Let, $y = zx$

$$\Rightarrow \quad \frac{dy}{dx}=z+x\frac{dz}{dx} \Rightarrow \frac{2z}{1+z^2}dz=-\frac{dx}{x}$$

Integrating $\int \frac{2z}{1+z^2}dz=-\int \frac{dx}{x}$

$\Rightarrow$ $\log(1 + z^2) = -\log x + c \Rightarrow (x^2 + y^2) = x \cdot e^c$

This curve passes through (1, 1)

$\Rightarrow$ $1 + 1 = 1 \cdot e^c$

$e^c = 2$

The required equation of the curve is

$\Rightarrow$ $x^2 + y^2 = 2x$

6. (3) : $y=u^m \Rightarrow \frac{dy}{dx}=mu^{m-1}\frac{du}{dx}$,

Hence $2x^4 \cdot u^m \cdot mu^{m-1} \cdot \frac{du}{dx}=u^{4m}+4x^6$

$$\frac{du}{dx}=\frac{4x^6-u^{4m}}{2mx^4u^{2m-1}} \Rightarrow 4m=6 \Rightarrow m=\frac{3}{2}$$

7. (0) : $(2ny + xy\log_e x)dx = x\log_e x dy$

$$\Rightarrow \quad \frac{dy}{y}=\left(\frac{2n}{x\log_e x}+1\right)dx$$

$\Rightarrow$ $\log(y) = 2n\log|\log x| + x + c$ and $c = 0$

Now, $g(x)=0.020\lim\limits_{n\to\infty} f(x)$

$$=\begin{cases} \to \quad \infty & \text{if} \quad x<\frac{1}{e} \\ 0 & \text{if} \quad \frac{1}{e}<x \quad <e \\ \to \quad \infty & \text{if} \quad x>e \end{cases} \qquad \therefore \int_{1/e}^{e} g(x)\,dx=0$$

8. (a, b, c, d) : Equation of the normal at a point $P(x, y)$ is given by

$$Y-y=-\frac{1}{dy/dx}(X-x) \qquad \text{...(1)}$$

Let the point Q at the x-axis be $(x_1, 0)$. From (1) we get

$$y\frac{dy}{dx}=x_1-x \qquad \text{...(2)}$$

Now given that $PQ^2 = k^2$

We have $(x - x_1)^2 + y^2 = k^2$

or $x-x_1=\pm\sqrt{k^2-y^2}$

Hence using (2) we obtain $y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$ ...(3)

(3) is the required differential equation for such curves

Now solving (3) we get $\int \frac{-ydy}{\sqrt{k^2-y^2}}=\int -dx$

or $x^2 + y^2 = k^2$ which passes through $(0,k)$ & $(k, 0)$

9. (a, b, c) : Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we get

$$\left(\frac{dy}{dx}\right)^2=\frac{x}{a} \Rightarrow dy=\pm\left(\frac{x^{1/2}}{a^{1/2}}\right)dx$$

Integrating we get $y+c=\pm\frac{2}{3a^{1/2}}x^{3/2}$

$\Rightarrow$ $9a(y + c)^2 = 4x^3$

10. (a, b) : $\frac{xdx+ydy}{ydx-xdy}=\frac{x\sin^2(x^2+y^2)}{y^3}$

$$\Rightarrow \quad \frac{1}{2}\operatorname{cosec}^2(x^2+y^2)d(x^2+y^2)=\frac{x}{y}d\left(\frac{x}{y}\right)$$

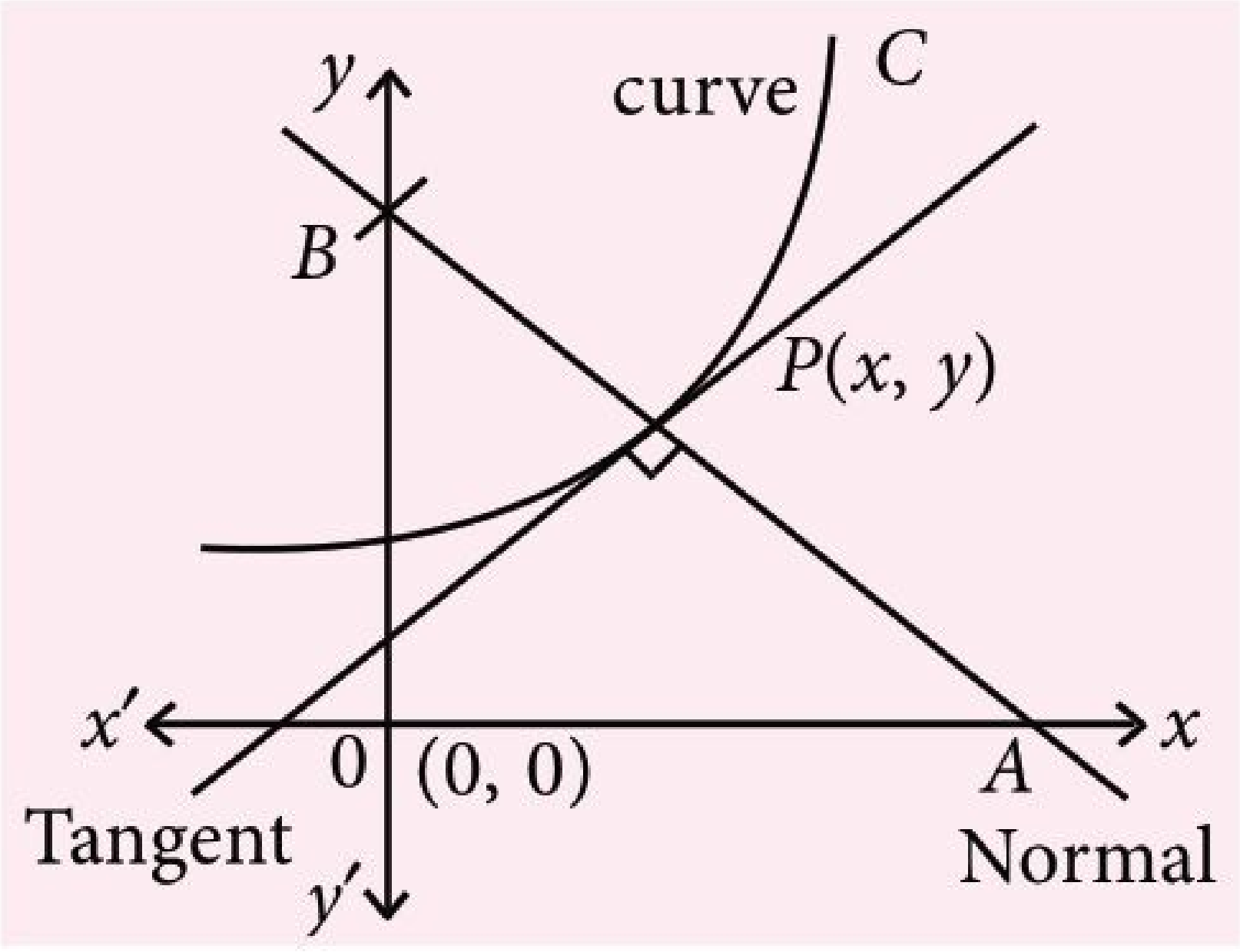
$$\Rightarrow \quad -\frac{1}{2}\cot(x^2+y^2)=\frac{1}{2}\left(\frac{x}{y}\right)^2+C$$

$$\Rightarrow \quad -\cot(x^2+y^2)=\frac{x}{y^2}^{2}+K$$

$$\Rightarrow \quad \frac{y^2}{x^2+ky^2}=-\tan(x^2+y^2)$$

11. (a, d) : The equation of normal of $P(x, y)$ is

$$(Y-y)=\frac{-1}{\frac{dy}{dx}}(X-x)$$



$\therefore$ $A\left(x+y\frac{dy}{dx}, 0\right)$ and

$B\left(0, y+\frac{x}{dy/dx}\right)$

Now, $\frac{1\left(x+y\frac{dy}{dx}\right)+2(0)}{1+2}=x$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(b)	**2.**	(a)	**3.**	(a)	**4.**	(a)	**5.**	(d)
6.	(c)	**7.**	(a,d)	**8.**	(a,b,d)	**9.**	(b,c,d)		
10.	(a,b,d)	**11.**	(b)	**12.**	(a,b,c)	**13.**	(b)	**14.**	(d)
15.	(b)	**16.**	(b)	**17.**	(2)	**18.**	(1)	**19.**	(1)
20.	(0.472)								

$$y\frac{dy}{dx}=2x \qquad \text{...(1)}$$

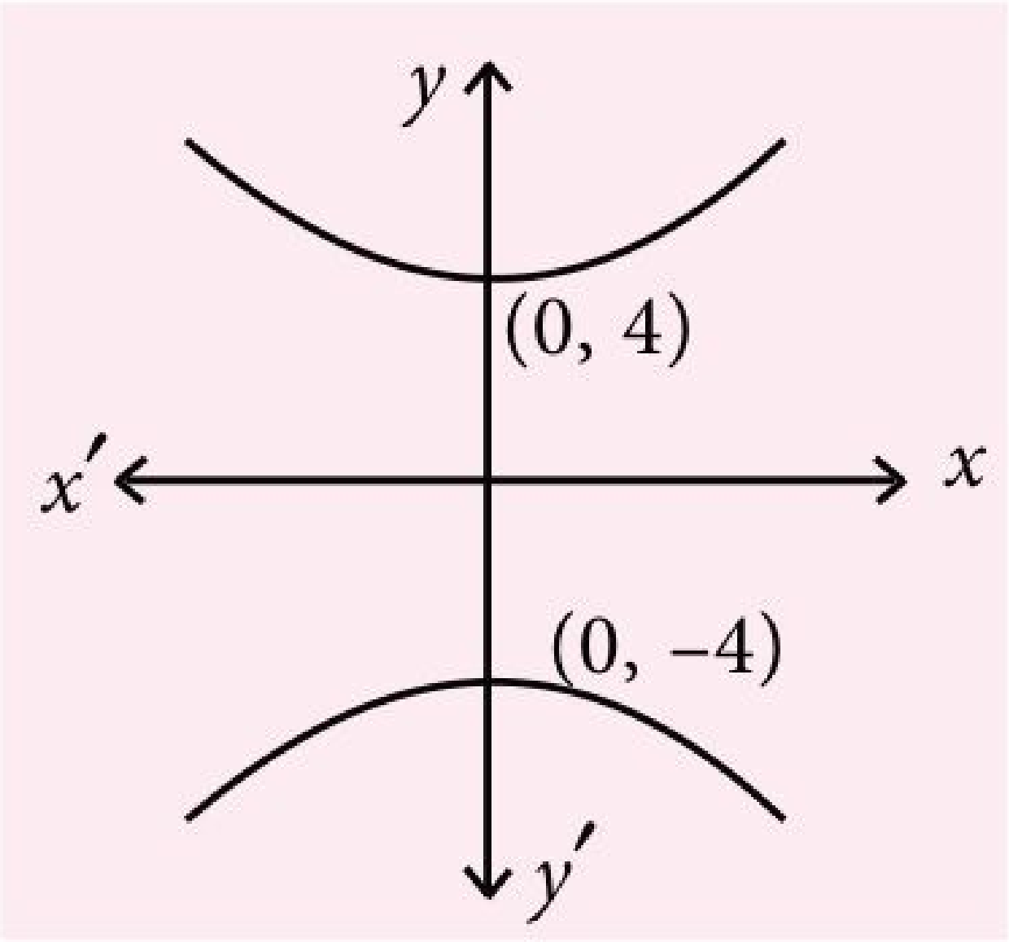
$$\Rightarrow \int y\,dy=\int 2x\,dx \Rightarrow \frac{y^2}{2}=x^2+C$$

Also (0, 4) satisfy it, so $C = 8$

$\therefore \quad y^2 = 2x^2 + 16$

(equation of curve)

Which represent a hyperbola



$$\text{Also, } \left.\frac{dy}{dx}\right]_{(4,\ 4\sqrt{3})}=\frac{2(4)}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$$

12. (a, c)

13. (a, b, c) : Given, $\frac{d}{dx}\left(y\frac{dy}{dx}\right)=x \Rightarrow y\frac{dy}{dx}=\frac{x^2}{2}+c$

$$\Rightarrow \frac{d}{dx}\left(\frac{y^2}{2}\right)=\frac{x^2}{2}+c$$

$$\Rightarrow y^2=\frac{x^3}{3}+\alpha x+\beta \text{ given } y(0)=1,\ y(1)=1$$

$$\Rightarrow \beta=1,\ \alpha=\frac{-1}{3}$$

$\therefore \quad y^2=f(x)=\frac{x^3-x+3}{3}$, $f'(x) = 3x^2 - 1 > 0$ for $x > 1$

$\therefore \quad f(x)\uparrow$

$$f\left(\frac{1}{\sqrt{3}}\right)f\left(-\frac{1}{\sqrt{3}}\right)>0$$

$\therefore \quad f(x) = 0$ has only one real root.

14. (b, d) : From the given equations we get

$x=\frac{t^2}{3}+\frac{t^3}{9}$ and $y=\frac{t^2}{6}-\frac{t^3}{9}$.

Eliminating t gives $8(x + y)^3 = 9(x - 2y)^2$

15. (b, c) : Dividing throughout by $\cos^2 x \sin^2 y$, the given differential equation becomes

$(3 \tan x + 4 \cot y - 7)\sec^2 x\, dx -$

$(4 \tan x + 7 \cot y - 5) \operatorname{cosec}^2 y\, dy = 0$

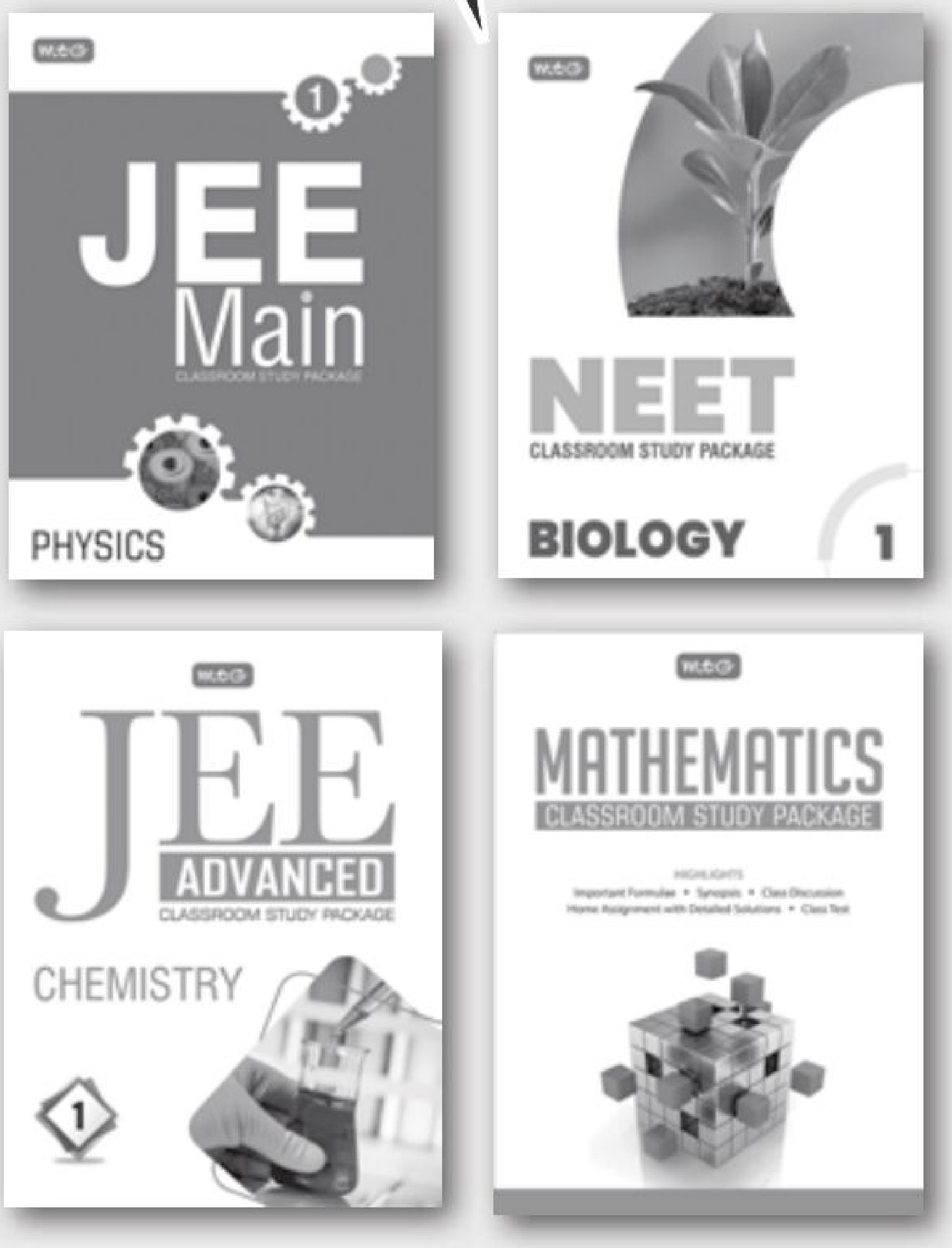
$\Rightarrow 3 \tan x \sec^2 x\, dx - 7 \sec^2 x\, dx - 7 \cot y \operatorname{cosec}^2 y\, dy +$ $5 \operatorname{cosec}^2 y\, dy + 4 \cot y \sec^2 x\, dx - 4 \tan x \operatorname{cosec}^2 y\, dy = 0$

$\Rightarrow 3 \tan x\, d(\tan x) - 7 d(\tan x) + 7 \cot y\, d(\cot y) - 5\, d(\cot y) + 4 d(\tan x \cot y) = 0$

On integrating, we obtain

$$\frac{3}{2}\tan^2 x-7\tan x+\frac{7}{2}\cot^2 y-5\cot y+4\tan x\cot y=C.$$

◆ ◆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SOLVED PAPER 2022

Category-I (Q.1 to 50)

(Carry 1 mark each. Only one option is correct. Negative marks : ¼)

1. Under which of the following condition(s) does(do) the system of equations $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & (a-4) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 6 \\ 4 \\ a \end{pmatrix}$ possesses(posses) unique solution?

(a) $\forall\ a \in R$ (b) $a = 8$

(c) for all integral values of a

(d) $a \neq 8$

2. If $\Delta(x)=\begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$, then coefficient of x in $\Delta(x)$ is

(a) 2 (b) –2 (c) 3 (d) –4

3. If $A=\begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$ and $A^{2018}=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $(a + d)$ equals

(a) $1 + i$ (b) 0 (c) 2 (d) 2018

4. Let S, T, U be three non-void sets and $f : S \to T$, $g : T \to U$ and composed mapping $g{\cdot}f : S \to U$ be defined. Let $g{\cdot}f$ be injective mapping. Then

(a) f, g both are injective

(b) neither f nor g is injective

(c) f is obviously injective

(d) g is obviously injective

5. If $p=\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of the 3 × 3 matrix A and det $A = 4$, then α is equal to

(a) 4 (b) 11 (c) 5 (d) 0

6. A, B, C are mutually exclusive events such that $P(A)=\dfrac{3x+1}{3}$, $P(B)=\dfrac{1-x}{4}$ and $P(C)=\dfrac{1-2x}{2}$. Then the set of possible values of x are in

(a) $[0, 1]$ (b) $\left[\frac{1}{3},\frac{1}{2}\right]$ (c) $\left[\frac{1}{3},\frac{2}{3}\right]$ (d) $\left[\frac{1}{3},\frac{13}{3}\right]$

7. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen is non-zero is

(a) $\frac{3}{16}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) $\frac{5}{8}$

8. For the mapping $f : R - \{1\} \to R - \{2\}$, given by $f(x)=\dfrac{2x}{x-1}$, which of the following is correct?

(a) f is one-one but not onto

(b) f is onto but not one-one

(c) f is neither one-one nor onto

(d) f is both one-one and onto

9. If the algebraic sum of the distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero, then the line passes through the fixed point

(a) (–1, 1) (b) (1, –1) (c) (–1, –1) (d) (1, 1)

10. The side AB of ΔABC is fixed and is of length $2a$ unit. The vertex moves in the plane such that the vertical angle is always constant and is α. Let x-axis be along AB and the origin be at A. Then the locus of the vertex is

(a) $x^2 + y^2 + 2ax \sin\alpha + a^2 \cos\alpha = 0$

(b) $x^2 + y^2 - 2ax - 2ay \cot\alpha = 0$

(c) $x^2 + y^2 - 2ax \cos\alpha - a^2 = 0$

(d) $x^2 + y^2 - ax \sin\alpha - ay \cos\alpha = 0$

11. If $(\cot\alpha_1)(\cot\alpha_2)\ldots.(\cot\alpha_n)=1, 0<\alpha_1,\alpha_2,\ldots,\alpha_n<\pi/2$, then the maximum value of $(\cos\alpha_1)(\cos\alpha_2)\ldots\ldots.(\cos\alpha_n)$ is given by

(a) $\frac{1}{2^{n/2}}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2n}$ (d) 1

12. A line passes through the point (–1, 1) and makes an angle $\sin^{-1}\left(\frac{3}{5}\right)$ in the positive direction of x-axis. If this line meets the curve $x^2 = 4y - 9$ at A and B, then $|AB|$ is equal to

(a) $\frac{4}{5}$ unit (b) $\frac{5}{4}$ unit (c) $\frac{3}{5}$ unit (d) $\frac{5}{3}$ unit

13. Two circles $S_1 = px^2 + py^2 + 2g'x + 2f'y + d = 0$ and $S_2 = x^2 + y^2 + 2gx + 2fy + d' = 0$ have a common chord PQ. The equation of PQ is

(a) $S_1 - S_2 = 0$ (b) $S_1 + S_2 = 0$
(c) $S_1 - pS_2 = 0$ (d) $S_1 + pS_2 = 0$

14. If the sum of the distance of a point from two perpendicular lines in a plane is 1 unit, then its locus is

(a) a square (b) a circle
(c) a straight line (d) two intersecting lines

15. Let P be a point on (2, 0) and Q be a variable point on $(y - 6)^2 = 2(x - 4)$. Then the locus of mid-point of PQ is

(a) $y^2 + x + 6y + 12 = 0$ (b) $y^2 - x + 6y + 12 = 0$
(c) $y^2 + x - 6y + 12 = 0$ (d) $y^2 - x - 6y + 12 = 0$

16. AB is a chord of parabola $y^2 = 4ax$, $(a > 0)$ with vertex A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

(a) a unit (b) $2a$ unit (c) $8a$ unit (d) $4a$ unit

17. Let $P(3 \sec\theta, 2 \tan\theta)$ and $Q(3 \sec\phi, 2 \tan\phi)$ be two points on $\frac{x^2}{9} - \frac{y^2}{4} = 1$ such that $\theta + \phi = \frac{\pi}{2}, 0 < \theta, \phi < \frac{\pi}{2}$. Then the ordinate of the point of intersection of the normals at P and Q is

(a) $\frac{13}{2}$ (b) $-\frac{13}{2}$ (c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

18. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to the x-axis is

(a) $y + 3z + 6 = 0$ (b) $y + 3z - 6 = 0$
(c) $y - 3z + 6 = 0$ (d) $y - 3z - 6 = 0$

19. The line $x - 2y + 4z + 4 = 0$, $x + y + z - 8 = 0$ intersect the plane $x - y + 2z + 1 = 0$ at the point

(a) (–2, 5, 1) (b) (2, –5, 1)
(c) (2, 5, –1) (d) (2, 5, 1)

20. AB is a variable chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If AB subtends a right angle at the origin O, then $\frac{1}{OA^2} + \frac{1}{OB^2}$ equals to

(a) $\frac{1}{a^2} + \frac{1}{b^2}$ (b) $\frac{1}{a^2} - \frac{1}{b^2}$
(c) $a^2 + b^2$ (d) $a^2 - b^2$

21. The values of a, b, c for which the function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \dfrac{(x + bx^2)^{1/2} - x^{1/2}}{bx^{1/2}}, & x > 0 \end{cases}$$

is continuous at $x = 0$, are

(a) $a = \frac{3}{2}, b = -\frac{3}{2}, c = \frac{1}{2}$

(b) $a = -\frac{3}{2}, c = \frac{3}{2}$, b is arbitrary non-zero real number.

(c) $a = -\frac{5}{2}, b = -\frac{3}{2}, c = \frac{3}{2}$

(d) $a = -2, b \in R - \{0\}, c = 0$

22. Let $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$, where a_0, a_1, a_2, a_3 are real constants. Then $f(x)$ is differentiable at $x = 0$

(a) whatever be a_0, a_1, a_2, a_3.
(b) for no values of a_0, a_1, a_2, a_3.
(c) only if $a_1 = 0$
(d) only if $a_1 = 0, a_3 = 0$

23. If $y = e^{\tan^{-1} x}$ then

(a) $(1 + x^2)y_2 + (2x - 1)y_1 = 0$
(b) $(1 + x^2)y_2 + 2xy = 0$
(c) $(1 - x^2)y_2 - y_1 = 0$
(d) $(1 + x^2)y_2 + 3xy_1 + 4y = 0$

24. Domain of $y = \sqrt{\log_{10} \frac{3x - x^2}{2}}$ is

(a) $x < 1$ (b) $2 < x$
(c) $1 \le x \le 2$ (d) $2 < x < 3$

25. $\lim_{x \to 0} \left(\frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}} \right)$ is

(a) 1/2 (b) 0
(c) 1 (d) does not exist

26. Let $f : [a, b] \to R$ be continuous in $[a, b]$, differentiable in (a, b) and $f(a) = 0 = f(b)$. Then

(a) there exists at least one point $c \in (a, b)$ for which $f'(c) = f(c)$
(b) $f'(x) = f(x)$ does not hold at any point of (a, b)
(c) at every point of (a, b), $f'(x) > f(x)$
(d) at every point of (a, b), $f'(x) < f(x)$

27. $I=\int \cos(\ln x)\,dx$. Then $I=$

(a) $\frac{x}{2}\{\cos(\ln x)+\sin(\ln x)\}+c$
(b) $x^2\{\cos(\ln x)-\sin(\ln x)\}+c$
(c) $x^2\sin(\ln x)+c$
(d) $x\cos(\ln x)+c$

28. Let f be derivable in $[0, 1]$, then
(a) there exists $c \in (0, 1)$ such that

$$\int_0^c f(x)dx=(1-c)f(c)$$

(b) there does not exist any point $d \in (0, 1)$ for which

$$\int_0^d f(x)dx=(1-d)f(d)$$

(c) $\int_0^c f(x)dx$ does not exist, for any $c \in (0, 1)$

(d) $\int_0^c f(x)dx$ is independent of c, $c \in (0, 1)$

29. The value of $\int_0^{\pi/2} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}}dx$ is

(a) $\frac{\pi}{4}$ (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

30. Let $\lim_{\in\to 0^+}\int_{\in}^{x}\frac{bt\cos 4t-a\sin 4t}{t^2}dt=\frac{a\sin 4x}{x}-1$, $\left(0<x<\frac{\pi}{4}\right)$. Then a and b are given by

(a) $a=2, b=2$ (b) $a=\frac{1}{4}, b=1$
(c) $a=-1, b=4$ (d) $a=2, b=4$

31. Let $\int\frac{x^{1/2}}{\sqrt{1-x^3}}dx=\frac{2}{3}g(f(x))+c$; then

(a) $f(x)=\sqrt{x},\ g(x)=x^{3/2}$
(b) $f(x)=x^{3/2},\ g(x)=\sin^{-1}x$
(c) $f(x)=\sqrt{x},\ g(x)=\sin^{-1}x$
(d) $f(x)=\sin^{-1}x,\ g(x)=x^{3/2}$

32. If $x\frac{dy}{dx}+y=x\frac{f(xy)}{f'(xy)}$, then $|f(xy)|$ is equal to

(a) $Ce^{\frac{x^2}{2}}$ (b) Ce^{x^2} (c) Ce^{2x^2} (d) $Ce^{\frac{x^2}{3}}$
where C is the constant of integration.

33. A curve passes through the point (3, 2) for which the segment of the tangent line contained between the co-ordinate axes is bisected at the point of contact. The equation of the curve is

(a) $y=x^2-7$ (b) $x=\frac{y^2}{2}+2$
(c) $xy=6$
(d) $x^2+y^2-5x+7y+11=0$

34. Let $f(x)=\int_{\sin x}^{\cos x}e^{-t^2}dt$. Then $f'\left(\frac{\pi}{4}\right)$ equals

(a) $\sqrt{\frac{1}{e}}$ (b) $-\sqrt{\frac{2}{e}}$ (c) $\sqrt{\frac{2}{e}}$ (d) $-\sqrt{\frac{1}{e}}$

35. The point of contact of the tangent to the parabola $y^2=9x$ which passes through the point (4, 10) makes an angle θ with the positive side of the axis of the parabola where $\tan\theta>2$, is

(a) $\left(\frac{4}{9},2\right)$ (b) $(4, 6)$
(c) $(4, 5)$ (d) $\left(\frac{1}{4},\frac{1}{6}\right)$

36. Let $f(x)=(x-2)^{17}(x+5)^{24}$. Then
(a) f does not have a critical point at $x=2$
(b) f has a minimum at $x=2$
(c) f has neither a maximum nor a minimum at $x=2$
(d) f has a maximum at $x=2$

37. The solution of $\cos y\frac{dy}{dx}=e^{x+\sin y}+x^2e^{\sin y}$ is $f(x)+e^{-\sin y}=C$ (C is arbitrary real constant) where $f(x)$ is equal to

(a) $e^x+\frac{1}{2}x^3$ (b) $e^{-x}+\frac{1}{3}x^3$
(c) $e^{-x}+\frac{1}{2}x^3$ (d) $e^x+\frac{1}{3}x^3$

38. Area of the figure bounded by the parabola $y^2+8x=16$ and $y^2-24x=48$ is

(a) $\frac{11}{9}$ sq. unit (b) $\frac{32}{3}\sqrt{6}$ sq. unit
(c) $\frac{16}{3}$ sq. unit (d) $\frac{24}{5}$ sq. unit

39. A particle moving in a straight line starts from rest and the acceleration at any time t is $a - kt^2$ where a and k are positive constants. The maximum velocity attained by the particle is

(a) $\frac{2}{3}\sqrt{\frac{a^3}{k}}$ (b) $\frac{1}{3}\sqrt{\frac{a^3}{k}}$ (c) $\sqrt{\frac{a^3}{k}}$ (d) $2\sqrt{\frac{a^3}{k}}$

40. If $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ is unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

(a) $\pm\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$ (b) $\pm\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

(c) $\pm\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (d) $\pm\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

41. If the equation of one tangent to the circle with centre at $(2, -1)$ from the origin is $3x + y = 0$, then the equation of the other tangent through the origin is
(a) $3x - y = 0$ (b) $x + 3y = 0$
(c) $x - 3y = 0$ (d) $x + 2y = 0$

42. If a, b, c are in G.P. and $\log a - \log 2b$, $\log 2b - \log 3c$, $\log 3c - \log a$ are in A.P., then a, b, c are the lengths of the sides of a triangle which is
(a) acute angled (b) obtuse angled
(c) right angled (d) equilateral

43. Let $a_n = (1^2 + 2^2 + \ldots\ldots n^2)^n$ and $b_n = n^n (n!)$. Then
(a) $a_n < b_n \ \forall\ n$
(b) $a_n > b_n \ \forall\ n$
(c) $a_n = b_n$ for infinitely many n
(d) $a_n < b_n$ if n be even and $a_n > b_n$ if n be odd

44. If $z = x - iy$ and $z^{1/3} = p + iq$ $(x, y, p, q \in R)$, then $\dfrac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to

(a) 2 (b) -1 (c) 1 (d) -2

45. If a, b are odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$ are
(a) rational (b) irrational
(c) non-real (d) equal

46. The number of zeros at the end of $\underline{|100}$ is

(a) 21 (b) 22 (c) 23 (d) 24

47. If $|z - 25i| \leq 15$, then maximum $\arg(z)$ – minimum $\arg(z)$ is equal to

(a) $2\cos^{-1}\left(\frac{3}{5}\right)$ (b) $2\cos^{-1}\left(\frac{4}{5}\right)$

(c) $\frac{\pi}{2}+\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)-\cos^{-1}\left(\frac{3}{5}\right)$

(arg z is the principal value of argument of z)

48. Let $f(n) = 2^{n+1}$, $g(n) = 1 + (n + 1)2^n$ for all $n \in N$. Then
(a) $f(n) > g(n)$
(b) $f(n) < g(n)$
(c) $f(n)$ and $g(n)$ are not comparable.
(d) $f(n) > g(n)$ if n be even and $f(n) < g(n)$ if n be odd.

49. A is a set containing n elements. P and Q are two subsets of A. Then the number of ways of choosing P and Q so that $P \cap Q = \phi$ is
(a) $2^{2n} - {}^{2n}C_n$ (b) 2^n
(c) $3^n - 1$ (d) 3^n

50. There are n white and n black balls marked 1, 2, 3, ..., n. The number of ways in which we can arrange these balls in a row so that neighbouring balls are of different colours is

(a) $(n!)^2$ (b) $(2n!)$ (c) $2(n!)^2$ (d) $\frac{(2n)!}{(n!)^2}$

Category-II (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : ½)

51. If P_1P_2 and P_3P_4 are two focal chords of the parabola $y^2 = 4ax$ then the chords P_1P_3 and P_2P_4 intersect on the
(a) directrix of the parabola
(b) axis of the parabola
(c) latus-rectum of the parabola
(d) y-axis

52. $f: X \rightarrow R$, $X = \{x \mid 0 < x < 1\}$ is defined as $f(x)=\dfrac{2x-1}{1-|2x-1|}$. Then

(a) f is only injective
(b) f is only surjective
(c) f is bijective
(d) f is neither injective nor surjective

53. $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$ and $f(0) = 0$. Then

(a) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$

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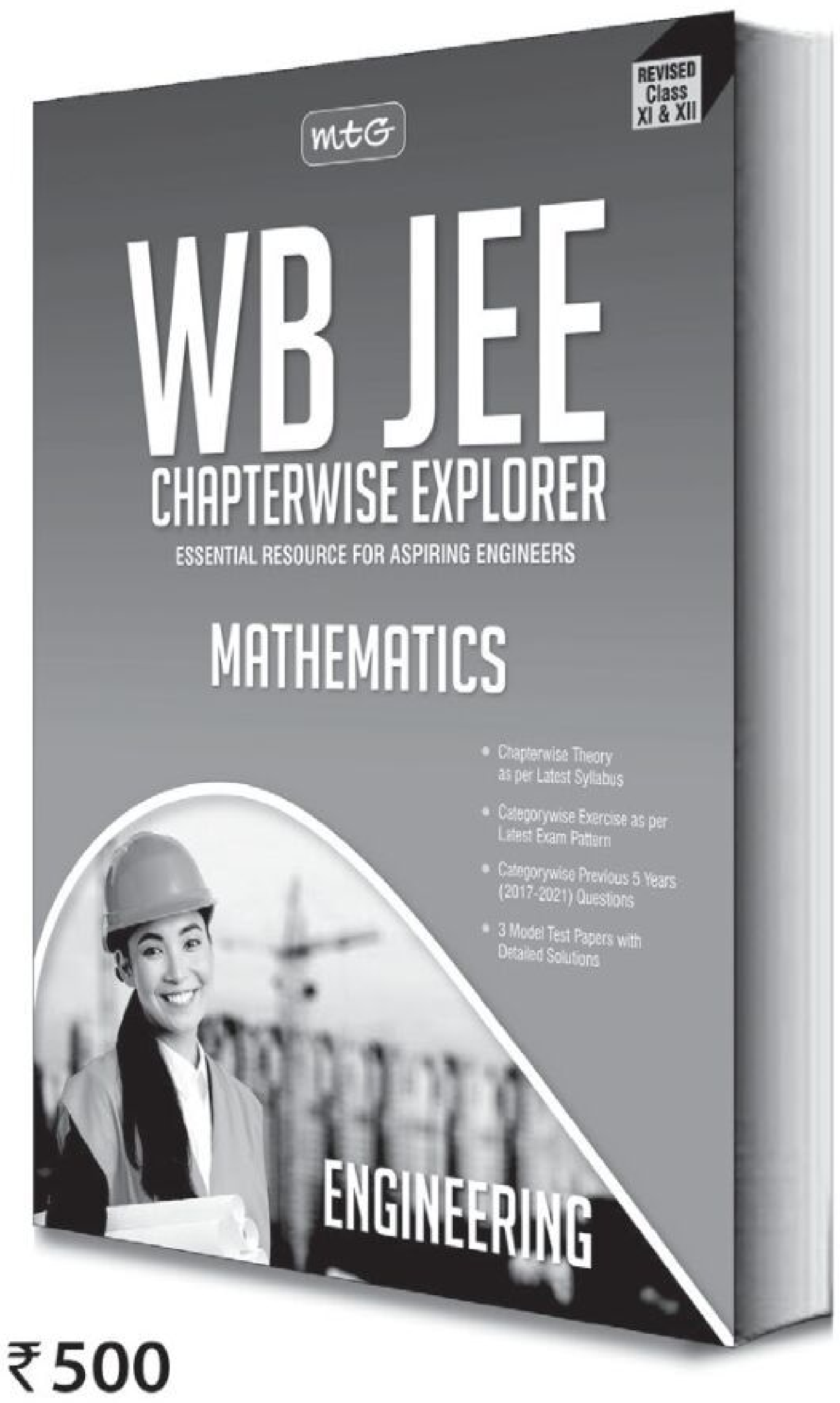
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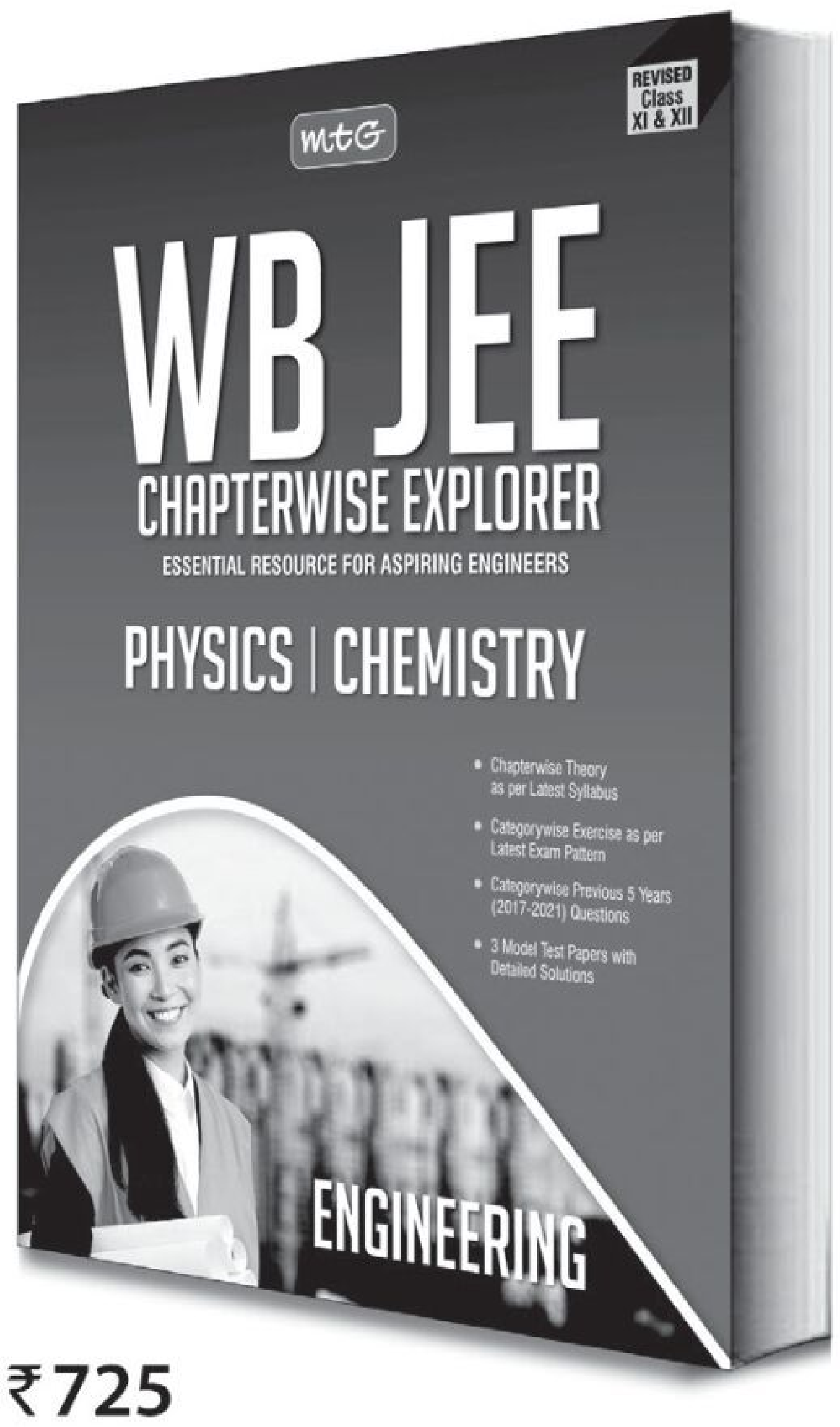
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(b) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

(c) $f\left(\frac{4}{3}\right)<\frac{4}{3}$ and $f\left(\frac{2}{3}\right)<\frac{2}{3}$

(d) $f\left(\frac{4}{3}\right)>\frac{4}{3}$ and $f\left(\frac{2}{3}\right)>\frac{2}{3}$

54. PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity e of the hyperbola satisfies

(a) $1<e<\frac{2}{\sqrt{3}}$ (b) $e=\frac{2}{\sqrt{3}}$

(c) $e=2\sqrt{3}$ (d) $e>\frac{2}{\sqrt{3}}$

55. $\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right),\ (a,b\in R)=0.$ Then

(a) $a=0, b=1$ (b) $a=1, b=-1$

(c) $a=-1, b=1$ (d) $a=0, b=0$

56. If the transformation $z=\log\tan\frac{x}{2}$ reduces the differential equation $\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+4y\operatorname{cosec}^2 x=0$ into the form $\frac{d^2y}{dz^2}+ky=0$ then k is equal to

(a) -4 (b) 4 (c) 2 (d) -2

57. If I is the greatest of

$$I_1=\int_0^1 e^{-x}\cos^2 x\,dx,\ I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx,$$

$$I_3=\int_0^1 e^{-x^2}dx,\ I_4=\int_0^1 e^{-x^2/2}\,dx,\text{ then}$$

(a) $I=I_1$ (b) $I=I_2$ (c) $I=I_3$ (d) $I=I_4$

58. A straight line meets the co-ordinate axes at A and B. A circle is circumscribed about the triangle OAB, O being the origin. If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is

(a) $m(m+n)$ (b) $m+n$

(c) $n(m+n)$ (d) $\frac{1}{2}(m+n)$

59. Let the tangent and normal at any point $P(at^2, 2at)$, $(a>0)$, on the parabola $y^2=4ax$ meet the axis of the parabola at T and G respectively. Then the radius of the circle through P, T and G is

(a) $a(1+t^2)$ (b) $(1+t^2)$

(c) $a(1-t^2)$ (d) $(1-t^2)$

60. From the point $(-1,-6)$, two tangents are drawn to $y^2=4x$. Then the angle between the two tangents is

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/2$

61. If $\vec{\alpha}$ is a unit vector, $\vec{\beta}=\hat{i}+\hat{j}-\hat{k}, \vec{\gamma}=\hat{i}+\hat{k}$, then the maximum value of $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]$ is

(a) 3 (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{6}$

62. The maximum value of $f(x)=e^{\sin x}+e^{\cos x}$; $x\in R$ is

(a) $2e$ (b) $2\sqrt{e}$ (c) $2e^{\frac{1}{\sqrt{2}}}$ (d) $2e^{\frac{-1}{\sqrt{2}}}$

63. If x satisfies the inequality $\log_{25}x^2+(\log_5 x)^2<2$, then x belongs to

(a) $\left(\frac{1}{5},5\right)$ (b) $\left(\frac{1}{25},5\right)$

(c) $\left(\frac{1}{5},25\right)$ (d) $\left(\frac{1}{25},25\right)$

64. The solution of $\det(A-\lambda I_2)=0$ be 4 and 8 and $A=\begin{pmatrix}2 & 3\\ x & y\end{pmatrix}$. Then

(a) $x=4, y=10$ (b) $x=5, y=8$

(c) $x=3, y=9$ (d) $x=-4, y=10$

65. The value of a for which the sum of the squares of the roots of the equation $x^2-(a-2)x-a-1=0$ assumes the least value is

(a) 0 (b) 1 (c) 2 (d) 3

Category-III (Q.66 to 75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. Chords of an ellipse are drawn through the positive end of the minor axis. Their midpoint lies on

(a) a circle (b) a parabola

(c) an ellipse (d) a hyperbola

67. Consider the equation $y-y_1=m(x-x_1)$. If m and x_1 are fixed an different lines are drawn for different values of y_1, then

(a) the lines will pass through a fixed point

(b) there will be a set of parallel lines

(c) all lines intersect the line $x=x_1$

(d) all lines will be parallel to the line $y=x_1$

68. Let R and S be two equivalence relations on a non-void set A. Then

(a) $R \cup S$ is equivalence relation

(b) $R \cap S$ is equivalence relation

(c) $R \cap S$ is not equivalence relation

(d) $R \cup S$ is not equivalence relation

69. Twenty metres of wire is available to fence off a flower bed in the form of a circular sector. What must the radius of the circle be, if the area of the flower bed be greatest?

(a) 10 m (b) 4 m (c) 5 m (d) 6 m

70. The line $y = x + 5$ touches

(a) the parabola $y^2 = 20x$

(b) the ellipse $9x^2 + 16y^2 = 144$

(c) the hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$

(d) the circle $x^2 + y^2 = 25$

71. Let $p(x)$ be a polynomial with real co-efficients, $p(0) = 1$ and $p'(x) > 0$ for all $x \in \mathbb{R}$. Then

(a) $p(x)$ has at least two real roots

(b) $p(x)$ has only one positive real root

(c) $p(x)$ may have negative real root

(d) $p(x)$ has infinitely many real roots

72. From a balloon rising vertically with uniform velocity v ft/sec a piece of stone is let go. The height of the balloon above the ground when the stone reaches the ground after 4 sec is $[g = 32 \text{ ft/sec}^2]$

(a) 220 ft (b) 240 ft (c) 256 ft (d) 260 ft

73. Let $f(x) = x^2 + x \sin x - \cos x$. Then

(a) $f(x) = 0$ has at least one real root

(b) $f(x) = 0$ has no real root

(c) $f(x) = 0$ has at least one positive root

(d) $f(x) = 0$ has at least one negative root

74. Let z_1 and z_2 be two non-zero complex numbers. Then

(a) Principal value of $\arg(z_1 z_2)$ may not be equal to Principal value of $\arg z_1$ + Principal value of $\arg z_2$

(b) Principal value of $\arg(z_1 z_2)$ = Principal value of $\arg z_1$ + Principal value of $\arg z_2$

(c) Principal value of $\arg(z_1/z_2)$ = Principal value of $\arg z_1$ – Principal value of $\arg z_2$

(d) Principal value of $\arg(z_1/z_2)$ may not be $\arg z_1 - \arg z_2$

75. Let $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$. Then

(a) Δ is independent of θ

(b) Δ is independent of ϕ

(c) Δ is a constant (d) $\left(\frac{d\Delta}{d\theta}\right)_{\theta = \pi/2} = 0$

SOLUTIONS

1. (d) : Given, $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & (a-4) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ a \end{pmatrix}$

Augmented matrix will be :

$$\left[\begin{array}{ccccc} 1 & 2 & 4 & \vdots & 6 \\ 2 & 1 & 2 & \vdots & 4 \\ 1 & 2 & (a-4) & \vdots & a \end{array}\right]$$

Applying $R_3 \to R_3 - R_1$, we get

$$\left[\begin{array}{ccccc} 1 & 2 & 4 & \vdots & 6 \\ 2 & 1 & 2 & \vdots & 4 \\ 0 & 0 & a-8 & \vdots & a-6 \end{array}\right]$$

For unique solution,

Rank of matrix A = Rank of matrix $[A|B]$ = Number of variables = 3

So, for $a \neq 8$ the system of equations has unique solution.

2. (b) : Given, $\Delta(x) = \begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$

$$\Rightarrow \Delta'(x) = \begin{vmatrix} 1 & (x-1)^2 & x^3 \\ 1 & x^2 & (x+1)^3 \\ 1 & (x+1)^2 & (x+2)^3 \end{vmatrix} + \begin{vmatrix} x-2 & 2(x-1) & x^3 \\ x-1 & 2x & (x+1)^3 \\ x & 2(x+1) & (x+2)^3 \end{vmatrix}$$

$$+ \begin{vmatrix} x-2 & (x-1)^2 & 3x^2 \\ x-1 & x^2 & 3(x+1)^2 \\ x & (x+1)^2 & 3(x+2)^2 \end{vmatrix}$$

$$\therefore \Delta'(0) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 8 \end{vmatrix} + \begin{vmatrix} -2 & -2 & 0 \\ -1 & 0 & 1 \\ 0 & 2 & 8 \end{vmatrix} + \begin{vmatrix} -2 & 1 & 0 \\ -1 & 0 & 3 \\ 0 & 1 & 12 \end{vmatrix}$$

$\Rightarrow \Delta'(0) = -2$

$\therefore$ Coefficient of x in $\Delta(x)$ is -2.

3. (b) : Given, $A = \begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix} \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix} \Rightarrow \quad A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also, $A^{2018} = A^{4 \times 504 + 2} = A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\therefore$ Using (i), we get

$a + d = 1 + (-1) = 0$

4. (c)

5. (b) : Given, $p = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

$|\text{adj } A| = |A|^{n-1}$

For $n = 3$, $|\text{adj } A| = |A|^2$

$\Rightarrow \quad |\text{adj } A| = 16 \qquad [\because |A| = 4]$

$$\text{Now, det} \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix} = 2\alpha - 6$$

$\Rightarrow \quad 2\alpha - 6 = 16 \Rightarrow \alpha = 11$

6. (b) : Given, $P(A) = \dfrac{3x+1}{3}$, $P(B) = \dfrac{1-x}{4}$,

$$P(C) = \frac{1-2x}{2}$$

$$\because \; 0 \le P \le 1 \Rightarrow \quad 0 \le \frac{3x+1}{3} \le 1 \Rightarrow -\frac{1}{3} \le x \le \frac{2}{3}$$

Also, $0 \le \dfrac{1-x}{4} \le 1 \Rightarrow -3 \le x \le 1$

Similarly, $0 \le \dfrac{1-2x}{2} \le 1 \Rightarrow -\dfrac{1}{2} \le x \le \dfrac{1}{2}$

Now, $0 \le P(A \cup B \cup C) \le 1$

$0 \le P(A) + P(B) + P(C) \le 1$

[$\because$ A, B, C are mutually exclusive events]

$$\Rightarrow \quad 0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$$

$$\Rightarrow \quad \frac{1}{3} \le x \le \frac{13}{3}$$

After comparison of inequalities of x and taking intersection, we get $\dfrac{1}{3} \le x \le \dfrac{1}{2}$

7. (b) : Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ using 0 or 1

$|A| \ne 0$ can be form in 6 ways as,

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}; \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

Also, the total number of ways of choosing a_{11}, a_{12}, a_{21}, and $a_{22} = 2 \times 2 \times 2 \times 2 = 16$

$\therefore$ Required probability $= \dfrac{6}{16} = \dfrac{3}{8}$

8. (d) : For one-one, $y = f(x) = \dfrac{2x}{x-1}, x \ne 1$

Suppose $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{2x_1}{x_1 - 1} = \frac{2x_2}{x_2 - 1} \quad \Rightarrow x_1 = x_2$$

So, f is one-one function.

For Range, $y = \dfrac{2x}{x-1}$

$$\Rightarrow \quad yx - y = 2x \Rightarrow \quad x = \frac{y}{y-2} \quad y \ne 2$$

$\therefore$ Range = Co-domain

So, f is also an onto function.

9. (d) : Let variable line be $ax + by + c = 0 \qquad \ldots\text{(i)}$

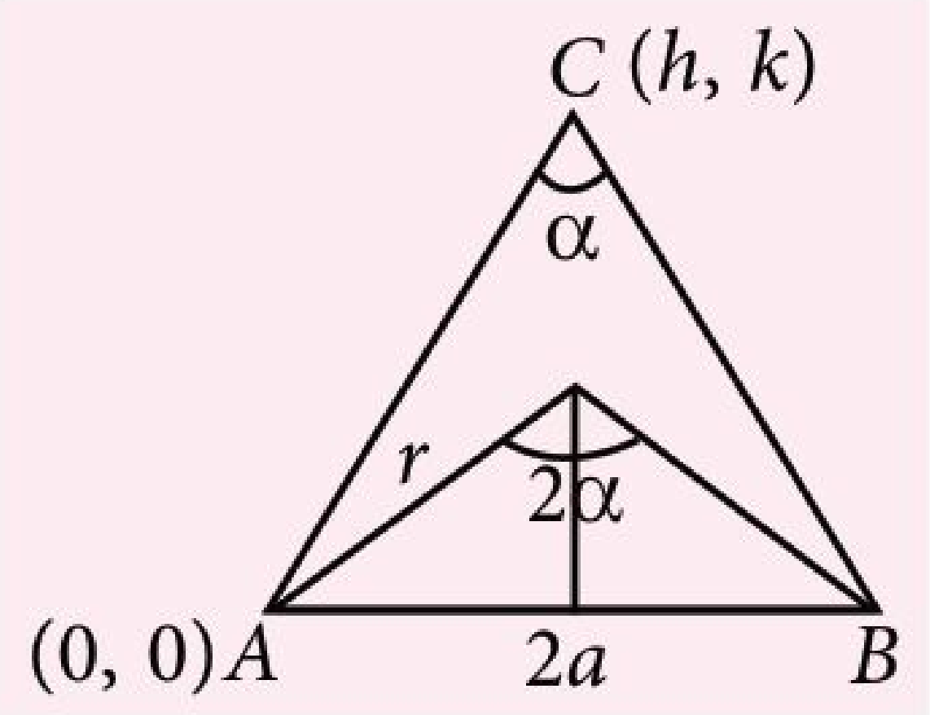
$$\therefore \left| \frac{2a + b \times 0 + c}{\sqrt{a^2+b^2}} \right| + \left| \frac{a \times 0 + b \times 2 + c}{\sqrt{a^2+b^2}} \right| + \left| \frac{a+b+c}{\sqrt{a^2+b^2}} \right| = 0$$

$$\Rightarrow \quad \pm\left(\frac{2a+c}{\sqrt{a^2+b^2}} \right) \pm \left(\frac{2b+c}{\sqrt{a^2+b^2}} \right) \pm \left(\frac{a+b+c}{\sqrt{a^2+b^2}} \right) = 0$$

$\Rightarrow \quad 2a + c + 2b + c + a + b + c = 0$

$\Rightarrow \quad 3a + 3b + 3c = 0 \Rightarrow a + b + c = 0$

Now, compare this with (i), we get, the coordinates of fixed point be (1, 1).



10. (b) : $\dfrac{a}{r} = \sin\alpha$

$\Rightarrow \quad r = a \operatorname{cosec}\alpha$

Centre $\equiv (a, a\cot\alpha)$

$(h - a)^2 + (k - a\cot\alpha)^2$

$\qquad = a^2 \operatorname{cosec}^2\alpha$

$\Rightarrow \quad h^2 + k^2 - 2ah - 2ak\cot\alpha = 0$

Locus of vertex is $x^2 + y^2 - 2ax - 2ay\cot\alpha = 0$

11. (a) : Given, $(\cot\alpha_1)(\cot\alpha_2) \cdots (\cot\alpha_n) = 1$

$\Rightarrow \quad (\cos\alpha_1)(\cos\alpha_2) \ldots (\cos\alpha_n) = \sin(\alpha_1)\sin(\alpha_2) \ldots \sin(\alpha_n)$

It is possible if, $\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_n = \dfrac{\pi}{4}$

$$\Rightarrow \quad \cos\alpha_1 \cdot \cos\alpha_2 \cdots \cos\alpha_n = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdots \cdot \frac{1}{\sqrt{2}}$$

$$= \left(\frac{1}{\sqrt{2}} \right)^n = \frac{1}{2^{n/2}}$$

12. (b) : Let equation of line is $y = mx + c$

Given, $\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$$\Rightarrow \sin\theta = \frac{3}{5} \Rightarrow \tan\theta = \frac{3}{4}$$

$\because$ Line passes through (–1, 1), so equation of line will be,

$$y = \frac{3}{4}x + \frac{7}{4} = \frac{1}{4}(3x+7)$$

$\because$ Line meets the curve $x^2 = 4y - 9$, Thus we get

$$x^2 = 4 \times \frac{1}{4}(3x+7) - 9$$

$\Rightarrow$ $x = 1, 2$

Point $A\left(1, \frac{10}{4}\right), B\left(2, \frac{13}{4}\right)$.

So, distance between AB is

$$|AB| = \sqrt{1^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4} \text{ unit}$$

13. (c) : Given,

$S_1 = px^2 + py^2 + 2g'x + 2f'y + d = 0$ ...(i)

$S_2 = x^2 + y^2 + 2gx + 2fy + d' = 0$

Make co-efficient of x^2 and y^2 in circle S_1 and S_2 equal so,

$pS_2 = p(x^2 + y^2 + 2gx + 2fy + d')$ ...(ii)

Subtract (ii) from (i), we get the equation of PQ *i.e.*,

$S_1 - pS_2 = 0$

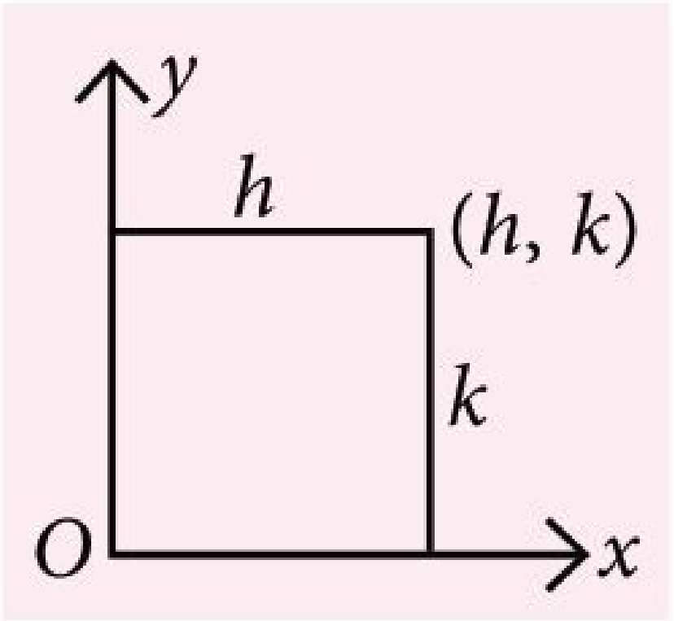
14. (a) : Assume x and y axis as perpendicular lines and (h, k)be any point. So,

$|h| + |k| = 1$

So, equation of locus will be,

$|x| + |y| = 1$

Thus, $|x| + |y|$ is a square.



15. (d) : Given $P(2, 0)$ and assume $Q(x, y)$ be any point on the curve $(y - 6)^2 = 2(x - 4)$

Let (h, k) be mid point of PQ. Then,

$h = \frac{2+x}{2} \Rightarrow x = 2(h - 1)$ ...(i)

and $k = \frac{0+y}{2} \Rightarrow 2k = y$

$\Rightarrow 2k = \pm\sqrt{2(x-4)} + 6$ ...(ii)

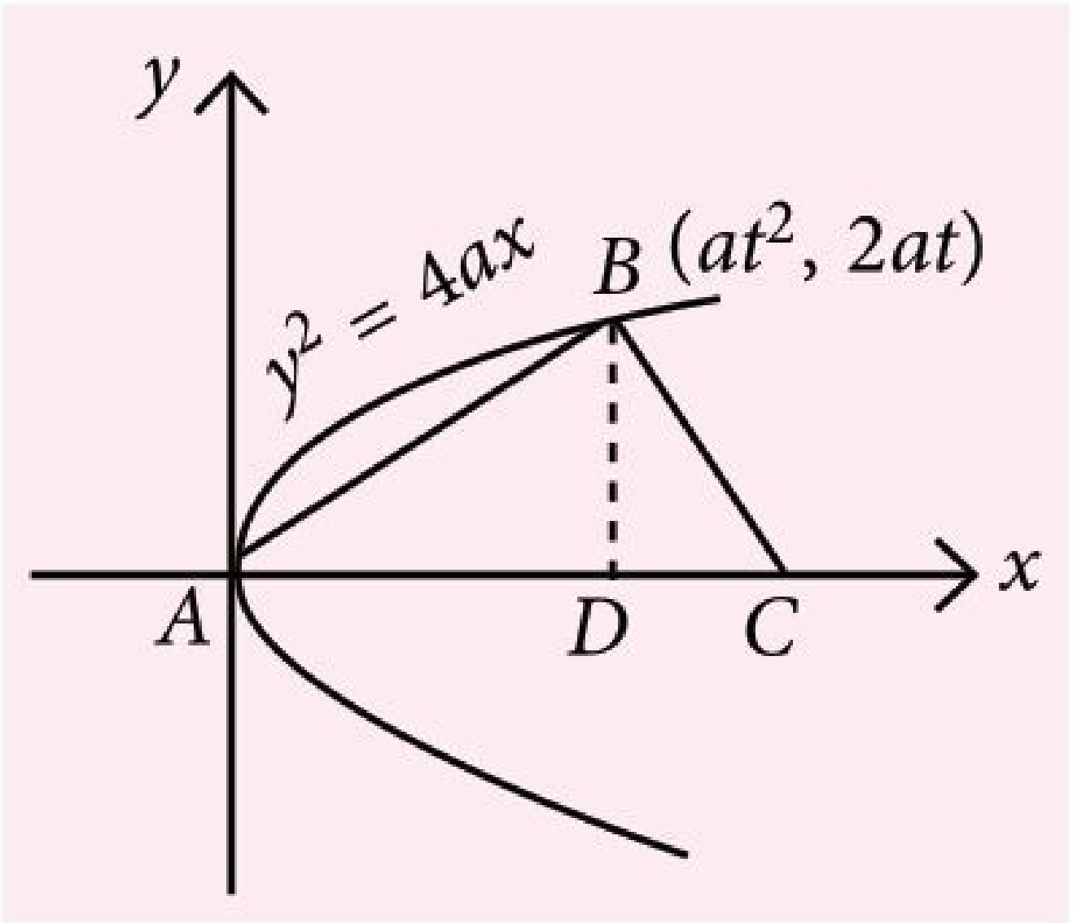
Substitute value of x from (i) in (ii), we get

$k^2 - h - 6k + 12 = 0$

So, locus of mid point of PQ is

$y^2 - x - 6y + 12 = 0$

16. (d) : Let point B is $(at^2, 2at)$



$\therefore$ Slope of line $AB = \frac{2}{t}$

$\therefore$ Slope of line $BC = -\frac{t}{2}$

$[\because BC \perp AB]$

Now, equation of line BC is

$$y - 2at = \frac{-t}{2}(x - at^2)$$

At C, $y = 0$ so, $x = 4a + at^2$

Length DC = projection of BC on axis of parabola

$= 4a + at^2 - at^2 = 4a$ unit

17. (b) : Given, $\frac{x^2}{9} - \frac{y^2}{4} = 1$

The equation of the normal at $(a\sec\theta, b\tan\theta)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $ax\cos\theta + by\cot\theta = a^2 + b^2$

Equation of normal at P is

$3x\cos\theta + 2y\cot\theta = 13$ ...(i)

Equation of normal at Q is

$3x\cos\phi + 2y\cot\phi = 13$

$\Rightarrow 3x\sin\theta + 2y\tan\theta = 13$ ...(ii) $\left(\because \theta + \phi = \frac{\pi}{2}\right)$

Solving equation (i) and (ii), we get

$$y = -\frac{13}{2}$$

18. (c) : Equation of plane will be

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) + (4\lambda - 1) = 0$ ...(i)

Since plane is parallel to x-axis.

$\therefore$ $1 + 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$

Put $\lambda = -\frac{1}{2}$ in (i), we get

$$-\frac{1}{2}y + \frac{3}{2}z - 3 = 0$$

$\Rightarrow$ $y - 3z + 6 = 0$

19. (d) : Equate the lines,

$x - 2y + 4z + 4 = x - y + 2z + 1$

$\Rightarrow$ $y - 2z = 3$ ...(i)

And, $x + y + z - 8 = x - y + 2z + 1$

$\Rightarrow$ $2y - z = 9$ ...(ii)

Solving (i) and (ii) we get,

$z = 1, y = 5$

Substituting $z = 1, y = 5$ in the plane $x - y + 2z + 1 = 0$, we get, $x = 2$

$\therefore$ Required point is (2, 5, 1).

20. (a)

21. (d) : Given, $f(x)=\begin{cases}\dfrac{\sin(a+1)\,x+\sin x}{x}, x<0\\ c, x=0\\ \dfrac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{1/2}}, x>0\end{cases}$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore$ L.H.L. = $f(0)$ = R.H.L.

$$\text{L.H.L.} = \underset{x\to 0}{\text{Lt}}\ \frac{\sin(a+1)\,x+\sin x}{x}\ \left(\frac{0}{0}\text{ form}\right)$$

$$= \underset{x\to 0}{\text{Lt}}\ \frac{(a+1)\cos(a+1)\,x+\cos x}{1}=a+2$$

$f(0) = c$

$$\text{R.H.L.} = \underset{x\to 0^+}{\text{Lt}}\ \frac{x^{1/2}[\sqrt{1+bx}-1]}{bx^{1/2}}=0$$

So, $a + 2 = c = 0 \Rightarrow a = -2$

The values of a, b, c for which $f(x)$ is continuous, are $a = -2$, $b \in R - \{0\}$, $c = 0$.

22. (c) : Given, $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$

We know that $|x|$ is not differentiable at $x = 0$.

$$\text{Let } f_2(x)=|x|^2=\begin{cases}x^2, & x<0\\ x^2, & x\ge 0\end{cases}$$

$$f_2'(x)=\begin{cases}2x, & x<0\\ 2x, & x\ge 0\end{cases}$$

So, $|x|^2$ is differentiable at $x = 0$.

Again, let

$$f_3(x)=|x|^3=\begin{cases}-x^3, & x<0\\ x^3, & x\ge 0\end{cases}$$

$$f_3'(x)=\begin{cases}-3x^2, & x<0\\ 3x^2, & x\ge 0\end{cases}$$

So, $|x|^3$ is differentiable at $x = 0$.

Hence, $f(x)$ is differentiable at $x = 0$ only if $a_1 = 0$.

23. (a) : We have, $y=e^{\tan^{-1}x}$

Differentiating w.r.t. x, we get

$$y_1=e^{\tan^{-1}x}\times\frac{1}{1+x^2}\ \Rightarrow\ y_1=\frac{y}{1+x^2}$$

Again, differentiating w.r.t. x, we get

$$y_2=\frac{(1+x^2)e^{\tan^{-1}x}\times\dfrac{1}{1+x^2}-e^{\tan^{-1}x}\times 2x}{(1+x^2)^2}$$

$$\Rightarrow\ y_2=\frac{y-2xy}{(1+x^2)^2}$$

$$\Rightarrow\ y_2=\frac{y}{(1+x^2)^2}-\frac{2xy}{(1+x^2)^2}=\frac{y_1}{(1+x^2)}-\frac{2xy_1}{(1+x^2)}$$

$\Rightarrow\ (1 + x^2)y_2 + (2x - 1)y_1 = 0$

24. (c) : Given, $y=\sqrt{\log_{10}\dfrac{3x-x^2}{2}}$

$\Rightarrow\ \dfrac{3x-x^2}{2}>0\ \Rightarrow\ x(3 - x) > 0$

$\Rightarrow\ x \in (0, 3)$ ...(i)

Also, $\log_{10}\left(\dfrac{3x-x^2}{2}\right)\ge 0$

$\Rightarrow\ x^2 - 3x + 2 \le 0$

$\Rightarrow\ x \in [1, 2]$ ...(ii)

From (i) and (ii), we get domain of y is $1 \le x \le 2$.

25. (c) : Given, $\displaystyle\lim_{x\to 0}\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}$

$$=\lim_{x\to 0}\frac{\ln\sqrt{1+x}-\ln\sqrt{1-x}}{x}$$

$$=\lim_{x\to 0}\frac{\frac{1}{2}\ln(1+x)-\frac{1}{2}\ln(1-x)}{x}\ \left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 0}\frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)=\lim_{x\to 0}\frac{1}{2}\times\left(\frac{1-x+1+x}{1-x^2}\right)=\frac{1}{2}\times 2=1$$

26. (a) : Let $g(x) = e^{-x}f(x)$

$\therefore\ g(a) = 0,\ g(b) = 0$

$\Rightarrow g(x)$ is continuous and differentiable. Thus, by Rolle's theorem,

$g'(c) = 0,\ c \in (a, b)$

$\Rightarrow\ e^{-x}f'(x) + (-e^{-x})f(x) = 0$

$\Rightarrow\ e^{-c}f'(c) = e^{-c}f(c)\ \Rightarrow f'(c) = f(c)$

27. (a) : $I=\int\cos(\ln x)\,dx=\int\cos(\ln x).1\,dx$

$$\Rightarrow\ I=\cos(\ln x).x-\int-\sin(\ln x)\cdot\frac{1}{x}\cdot x\,dx$$

$$\Rightarrow\ I=x\cos(\ln x)+\int\sin(\ln x)\cdot 1\,dx$$

$$\Rightarrow \quad I = x\cos(\ln x) + \left[\sin(\ln x)\cdot x - \int \cos(\ln x)\cdot \frac{1}{x}\cdot x\,dx\right]$$

$$\Rightarrow \quad I = x\cos(\ln x) + x\sin(\ln x) - I + c_1$$

$$\Rightarrow \quad 2I = x\big(\cos(\ln x) + \sin(\ln x)\big) + c_1$$

$$\Rightarrow \; I = \frac{x}{2}\big(\cos(\ln x) + \sin(\ln x)\big) + c$$

28. (a)

29. (a) : Let $I = \int_0^{\pi/2} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x} + (\sin x)^{\cos x}}\,dx \qquad \ldots\text{(i)}$

Also, $I = \int_0^{\pi/2} \frac{\left[\cos\left(\frac{\pi}{2}-x\right)\right]^{\sin\left(\frac{\pi}{2}-x\right)}}{\left[\cos\left(\frac{\pi}{2}-x\right)\right]^{\sin\left(\frac{\pi}{2}-x\right)} + \left[\sin\left(\frac{\pi}{2}-x\right)\right]^{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$

$$\Rightarrow \; I = \int_0^{\pi/2} \frac{(\sin x)^{\cos x}}{(\sin x)^{\cos x} + (\cos x)^{\sin x}}\,dx \qquad \ldots\text{(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{(\sin x)^{\cos x} + (\cos x)^{\sin x}}{(\sin x)^{\cos x} + (\cos x)^{\sin x}}\,dx$$

$$\Rightarrow \quad 2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$$

30. (b) : $f(x) = \lim_{\epsilon\to 0^+} \int_{\epsilon}^{x} \frac{bt\cos 4t - a\sin 4t}{t^2}\,dt = \frac{a\sin 4x}{x} - 1$

$$f'(x) = \frac{bx\cos 4x - a\sin 4x}{x^2} = \frac{4ax\cos 4x - a\sin 4x}{x^2} - 0$$

On equating, we get

$$\frac{bx\cos 4x - a\sin 4x}{x^2} = \frac{4ax\cos 4x - a\sin 4x}{x^2}$$

$\Rightarrow \quad b = 4a$

$\therefore$ Only option (b) satisfies the condition $b = 4a$.

31. (b) : Let $I = \int \frac{x^{\frac{1}{2}}}{\sqrt{1-x^3}}\,dx$

Let $z = x^{3/2} \;\Rightarrow\; dz = \frac{3}{2}x^{1/2}dx$

$$\therefore \quad I = \frac{2}{3}\int \frac{dz}{\sqrt{1-z^2}} \qquad \left[\because\; z^2 = x^3\right]$$

$$= \frac{2}{3}\sin^{-1} z + c = \frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right) + c$$

Hence, $g(x) = \sin^{-1}x$, $f(x) = x^{3/2}$

32. (a) : Given, $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)}$

$$\Rightarrow \quad \frac{d(xy)}{dx} = x\frac{f(xy)}{f'(xy)} \;\Rightarrow\; \int \frac{f'(xy)}{f(xy)}\,d(xy) = \int x\,dx$$

$$\Rightarrow \quad \ln f(xy) = \frac{x^2}{2} + c_1 \;\Rightarrow\; f(xy) = e^{\frac{x^2}{2} + c_1}$$

$$\therefore \quad |f(xy)| = Ce^{\frac{x^2}{2}}$$

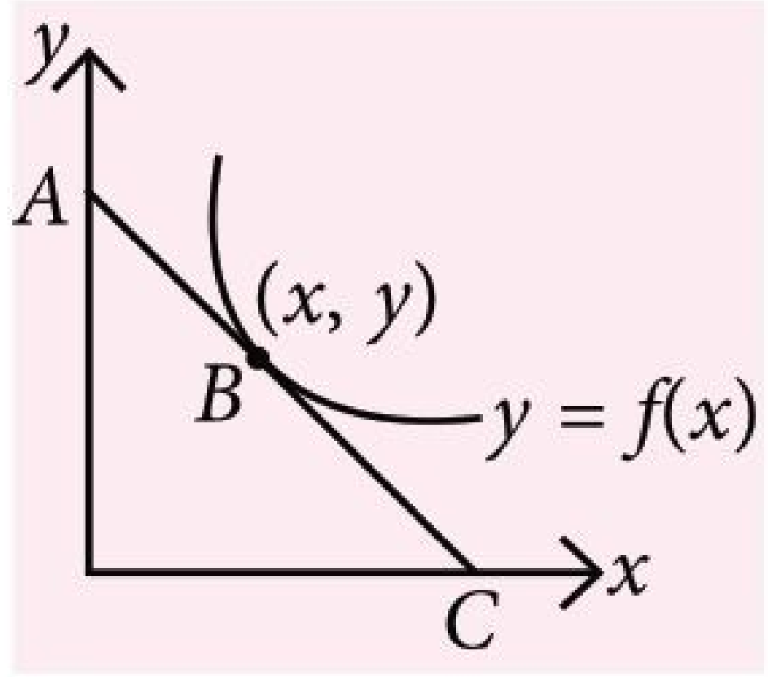
33. (c) : Given, $AB = BC$

Coordinate of point $A \equiv (0, 2y)$, $C \equiv (2x, 0)$

Slope of line AC is

$$\frac{dy}{dx} = \frac{-y}{x} \;\Rightarrow\; \frac{dy}{y} = -\frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{dy}{y} = -\int \frac{dx}{x}$$



$\Rightarrow \quad \ln y = -\ln x + \ln C \;\Rightarrow\; xy = C$

$\because$ Curve passes through (3, 2), so, $3 \times 2 = C$

$\Rightarrow \quad C = 6$

$\therefore$ The equation of curve is $xy = 6$.

34. (b) : $f'(x) = e^{-\cos^2 x}(-\sin x) - e^{-\sin^2 x}(\cos x)$

$$f'\left(\frac{\pi}{4}\right) = e^{-\frac{1}{2}}\left(-\frac{1}{\sqrt{2}}\right) - e^{-\frac{1}{2}}\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{\frac{2}{e}}$$

35. (a) : Given equation of parabola is $y^2 = 9x$

So, $a = \frac{9}{4}$

Any point on parabola be $P\left(\frac{9t^2}{4}, \frac{9t}{2}\right)$.

$\therefore \quad$ Equation of tangent at point P be $ty = x + \frac{9t^2}{4}$

$\because \quad$ The equation passes through (4, 10).

$\therefore \quad 40t = 16 + 9t^2 \;\Rightarrow\; t = \frac{4}{9}, 4$

But the slope is $\frac{1}{t}$ *i.e.*, $\frac{9}{4}$ or $\frac{1}{4}$.

Also, it is given that slope, $\tan\theta > 2$.

$\therefore \quad t = \frac{4}{9}$

Thus, point of contact is $\left(\frac{4}{9}, 2\right)$.

36. (c) : $f'(x) = (x-2)^{17} \times 24(x+5)^{23} + (x+5)^{24} \times 17(x-2)^{16}$

$= (x-2)^{16}(x+5)^{23}[24(x-2)+17(x+5)]$

Also $f''(x) = (x-2)^{15}(x+5)^{22}((x-2)(x+5).41 + 16(x+5)(41x+37) + 23(x-2)(41x+37))$

At $x = 2$, $f'(x)$ and $f''(x)$ do not change the sign.

So, f has neither a maximum nor a minimum at $x = 2$.

37. (d) : Given, $\cos y \cdot \frac{dy}{dx} = e^{x+\sin y} + x^2 e^{\sin y}$

$$\Rightarrow \cos y \cdot \frac{dy}{dx} = e^{\sin y}(e^x + x^2)$$

$$\Rightarrow \int e^{-\sin y}\cos y\, dy = \int (e^x + x^2)dx$$

$$\Rightarrow -e^{-\sin y} = e^x + \frac{x^3}{3} + C_1$$

$$\Rightarrow \left(e^x + \frac{x^3}{3}\right) + e^{-\sin y} = C$$

Hence, $f(x) = e^x + \frac{x^3}{3}$.

38. (b) : Given parabola $y^2 + 8x = 16$ and $y^2 - 24x = 48$ and the points of intersection are $(-1, -\sqrt{24})$ and $(-1, \sqrt{24})$

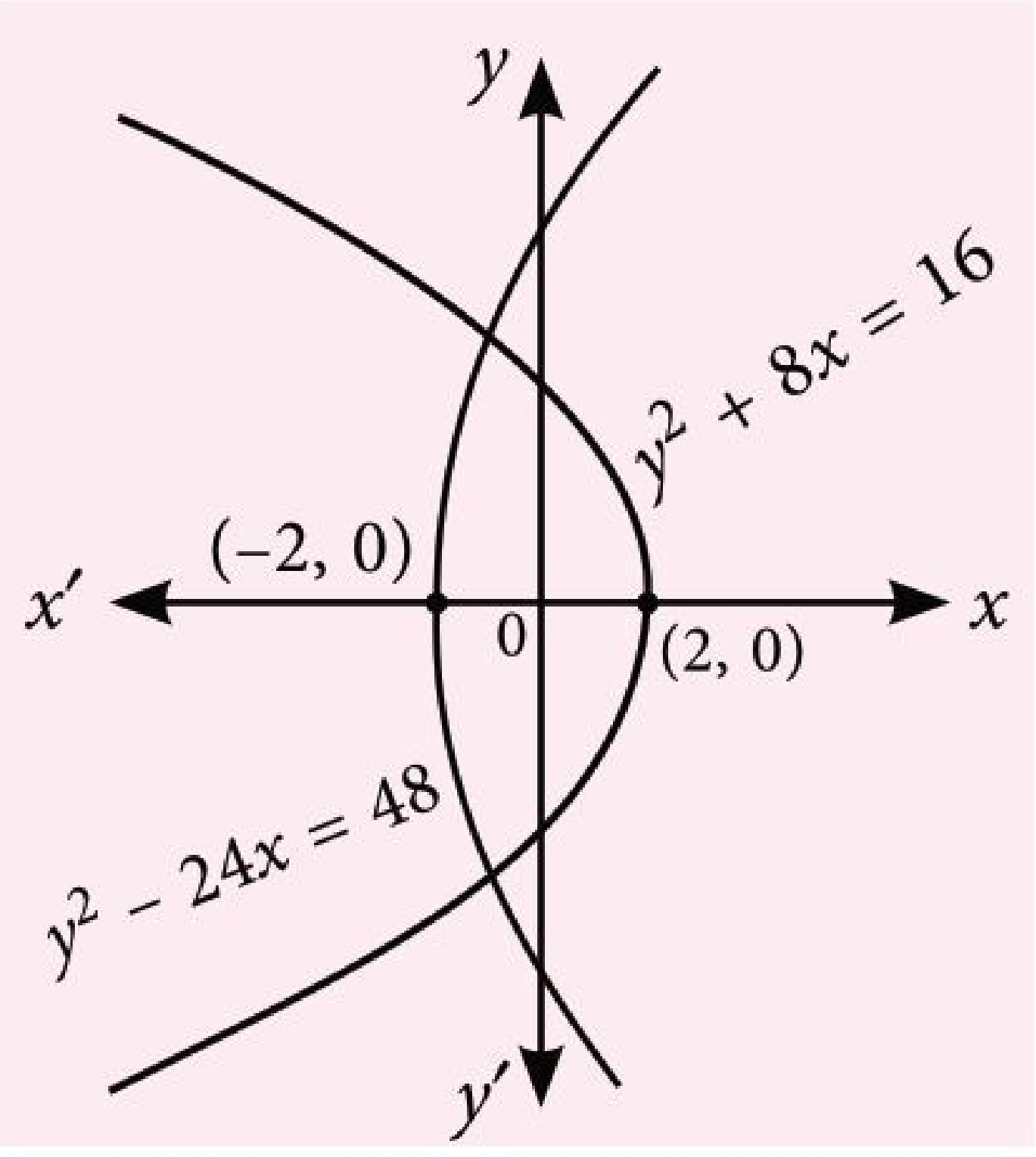
Area bounded by both the parabola is,

$$A = \int_{-\sqrt{24}}^{\sqrt{24}} \left(\frac{16-y^2}{8}\right) - \left(\frac{y^2-48}{24}\right)dy$$

$$= \int_{-\sqrt{24}}^{\sqrt{24}} \left(4 - \frac{1}{6}y^2\right)dy$$

$$= \left[4y - \frac{y^3}{18}\right]_{-\sqrt{24}}^{\sqrt{24}}$$

$$= \frac{32}{3}\sqrt{6} \text{ sq. unit}$$



39. (a) : Acceleration $= \frac{dv}{dt} = a - kt^2$

$$\Rightarrow \int dv = \int (a - kt^2)dt$$

$$\Rightarrow v = at - \frac{k}{3}t^3 + C$$

Since, particle starts from rest *i.e.*, at $t = 0$, $v = 0$

$\therefore$ $0 = 0 + C$

$\Rightarrow$ $C = 0$

$\therefore$ Velocity will be $v = t\left(a - \frac{k}{3}t^2\right)$ ...(i)

Also, velocity will be maximum when $\frac{dv}{dt} = 0$

i.e., $a - kt^2 = 0$

$\Rightarrow$ At $t = \sqrt{\frac{a}{k}}$, the velocity will be maximum and its value is $v_{\max} = \frac{2}{3}\sqrt{\frac{a^3}{k}}$ [Using (i)]

40. (b) : Given, $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Given $\vec{c}$ is perpendicular to $\vec{a}$. Therefore, $\vec{a}\cdot\vec{c} = 0$

Also, $|\vec{c}| = 1$

We also have, $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. Therefore, $\vec{c} = \vec{a} + \lambda\vec{b}$

$$\Rightarrow \vec{a}\cdot\vec{c} = \vec{a}\cdot(\vec{a} + \lambda\vec{b}) \Rightarrow 0 = |\vec{a}|^2 + \lambda\vec{a}\cdot\vec{b}$$

$$\Rightarrow 0 = \sqrt{3} + \lambda(1-1-1) \Rightarrow \lambda = \sqrt{3}$$

Now, $\vec{d} = \vec{a} \times \vec{c} = \vec{a} \times (\vec{a} + \lambda\vec{b}) = \sqrt{3}\,(\vec{a} \times \vec{b})$

$$= \sqrt{3}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \sqrt{3}(-2\hat{j} - 2\hat{k})$$

$$\therefore \hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{\sqrt{3}(-2\hat{j}-2\hat{k})}{|\sqrt{3}|\sqrt{0+4+4}} \Rightarrow \hat{d} = \pm\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$$

41. (c) : $\because$ Radius of circle = Length of ⊥ from centre to tangent

$$\Rightarrow r = \frac{3(2)-1}{\sqrt{9+1}} = \frac{5}{\sqrt{10}}$$

$\therefore$ Equation of circle is

$$(x-2)^2 + (y+1)^2 = \frac{25}{10} = \frac{5}{2} \quad \text{...(i)}$$

Let $y = mx$ be another tangent.

From (i), we get $(x-2)^2 + (mx+1)^2 = \frac{5}{2}$

$$\Rightarrow (1+m^2)x^2 + 4 - 4x + 1 + 2mx = \frac{5}{2}$$

$$\Rightarrow (1+m^2)x^2 + (2m-4)x + 5 = \frac{5}{2}$$

$$\Rightarrow (1+m^2)x^2 + (2m-4)x + \frac{5}{2} = 0$$

The above equation must have only one root because tangent touches the circle.

$\therefore$ Discriminant = 0

$$\Rightarrow (2m-4)^2-4(m^2+1)\left(\frac{5}{2}\right)=0$$

$$\Rightarrow (3m-1)(m+3)=0 \Rightarrow m=\frac{1}{3} \text{ or } m=-3$$

$\therefore$ Equation of another tangent is $y=\frac{1}{3}x$

$\Rightarrow x - 3y = 0$

42. (b) : Given a, b, c are in G.P.

$\therefore \quad b^2 = ac$ ...(i)

Also, we have

$2(\log 2b - \log 3c) = \log a - \log 2b + \log 3c - \log a$

$\Rightarrow \quad 2b = 3c$ ...(ii)

From (i) & (ii), we get

$$b=\frac{2a}{3} \text{ and } c=\frac{4a}{9}$$

Now, $a+b=\frac{5a}{3}>c$

$$b+c=\frac{10a}{9}>a \; ; \; c+a=\frac{13a}{9}>b$$

From above, we can say that a,b,c form a triangle where 'a' is greatest side.

By cosine rule, $\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{-29}{48}<0$

So, angle A is obtuse.

43. (b) : We have, $a_n = (1^2 + 2^2 + 3^2 + \ldots\ldots + n^2)^n$ and $b_n = n^n\,(n!)$

As, A.M. ≥ G.M.

$$\Rightarrow \frac{1^2+2^2+3^2+\ldots\ldots+n^2}{n} \geq \sqrt[n]{(1^2\cdot 2^2\cdot 3^2 \ldots\cdot n^2)}$$

$\Rightarrow 1^2 + 2^2 + 3^2 + \ldots\ldots n^2 \geq n(n!)^{2/n}$

Taking to the power of n, we get

$(1^2 + 2^2 + 3^2 + \ldots\ldots + n^2)^n \geq n^n(n!)^{2n/n} > n^n(n!)$ for $n \neq 1$

$\Rightarrow (1^2 + 2^2 + \ldots\ldots + n^2)^n > n^n(n!)$

$\Rightarrow a_n > b_n \; \forall n$

44. (d) : Given $z^{1/3}=p+iq$

$\Rightarrow z = (p + iq)^3 = (x - iy)$

$\Rightarrow x - iy = (p^3 - 3pq^2) + i(3p^2q - q^3)$

Comparing both sides, we get

$x = p^3 - 3pq^2$

$y = -3p^2q + q^3$

Now, $\frac{x}{p}+\frac{y}{q}=p^2-3q^2-3p^2+q^2=-2(p^2+q^2)$

$$\Rightarrow \frac{\frac{x}{p}+\frac{y}{q}}{(p^2+q^2)}=-2$$

45. (a) : We have, $2ax^2 + (2a + b)x + b = 0,\; a \neq 0$ ...(i)

Now, $D = (2a + b)^2 - 4(2a)b = (2a - b)^2$

$\therefore$ Roots of equation are given as

$$x=\frac{-(2a+b)\pm\sqrt{(2a-b)^2}}{4a}$$

$$=\frac{-2a-b+2a-b}{4a},\ \frac{-2a-b-2a+b}{4a}$$

$$=\frac{-b}{2a},-1$$

Hence, both roots are rational.

46. (d) : The number of zeros at the end of 100! is

$$\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]+\left[\frac{100}{5^3}\right]=20+4+0=24$$

47. (b)

48. (b) : $f(n) = 2^{n+1}$, $g(n) = 1 + (n + 1)2^n$

$g(n) - f(n) = 1 + n\cdot 2^n + 2^n - 2\cdot 2^n$

$= 1 + n\cdot 2^n - 2^n = 1 + (n - 1)2^n$

Since $n \in N$ so, for $n = 1$

$g(n) - f(n) = 1$

Hence, for all $n \geq 1$, $g(n) > f(n)$.

49. (d) : The number of ways for $P \cap Q = \phi$ is ${}^nC_0 \times 3^n = 3^n$

50. (c) : There are n white and n black balls.

The arrangement can be done as,

B W B W B W .... or W B W B W ....

So, n white balls can be arrange in $n!$ ways and n black balls can be arrange in $n!$ ways.

There are 2 possible arrangement. So,

Total ways = $2(n!)\,(n!) = 2(n!)^2$

51. (a) : Given equation of parabola $y^2 = 4ax$

Points of chord P_1P_2 are

$$P_1(at_1^2, 2at_1),\ P_2\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$$

Points of chord P_3P_4 are

$$P_3(at_2^2, 2at_2),\ P_4\left(\frac{a}{t_2^2}, \frac{-2a}{t_2}\right)$$

Equation of P_1P_3 is

$$\frac{x-at_1^2}{at_2^2-at_1^2}=\frac{y-2at_1}{2at_2-2at_1} \qquad \text{...(i)}$$

Equation of P_2P_4 is

$$2\left(x+\frac{a}{t_1t_2}\right)=-\left(\frac{1}{t_1}+\frac{1}{t_2}\right)y \qquad \text{...(ii)}$$

Solving equation (i) and (ii), we get

$x + a = 0$

which is equation of directrix of the parabola.

52. (a) : $y=\frac{2x-1}{1-|2x-1|}$

For injective: Let $x_1 \neq x_2$

$\Rightarrow \; 2x_1 \neq 2x_2 \Rightarrow 2x_1 - 1 \neq 2x_2 - 1$

$$\Rightarrow \frac{2x_1-1}{1-|2x_1-1|}\neq\frac{2x_2-1}{1-|2x_2-1|} \Rightarrow y_1 \neq y_2$$

So, f is injective.

Since, range is not co-domains, therefore f is not surjective.

Hence, f is only injective.

53. (c) : Given, $\int_0^x \sqrt{1-(f'(t))^2}\,dt=\int_0^x f(t)dt$

Differentiate both sides, we get

$\sqrt{1-(f'(x))^2}=f(x)$

So, $f(x) = \sin x$ or $\cos x$

Also, $f(0) = 0 \Rightarrow f(x) = \sin x$

So, $f\left(\frac{4}{3}\right)<\frac{4}{3}$ and $f\left(\frac{2}{3}\right)<\frac{2}{3}$

54. (d) : Given hyperbola is, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Point P and Q are $(a\sec\theta, b\tan\theta)$, $(a\sec\theta, -b\tan\theta)$

Since, ΔOPQ is an equilateral triangle.

$$\text{So, } \tan 30°=\frac{b\tan\theta}{a\sec\theta}$$

$$\Rightarrow \sin^2\theta=\frac{a^2}{3b^2} \Rightarrow \operatorname{cosec}^2\theta=\frac{3b^2}{a^2}$$

$\Rightarrow \operatorname{cosec}^2\theta = 3(e^2 - 1)$

Since, $\operatorname{cosec}^2\theta \geq 1$

Hence, $3(e^2 - 1) \geq 1$

$$\Rightarrow e^2 \geq \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$

55. (b) : $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)$

$$=\lim_{x\to\infty}\left[\frac{x^2+1-ax(x+1)-b(x+1)}{x+1}\right]$$

$$=\lim_{x\to\infty}\left[\frac{x^2(1-a)-x(a+b)+(1-b)}{x+1}\right]$$

This limit will be 0 when co-efficient of x^2 and x will be 0.

So, $1 - a = 0 \Rightarrow a = 1$

and $a + b = 0 \Rightarrow b = - a = -1$

56. (b) : $z=\log\tan\frac{x}{2}$

$$\frac{dz}{dx}=\frac{1}{2}\frac{\sec^2\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}=\operatorname{cosec}x$$

$$\text{Now, } \frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}=\operatorname{cosec}x\cdot\frac{dy}{dz} \qquad \text{...(i)}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\operatorname{cosec}^2x\frac{d^2y}{dz^2}-\operatorname{cosec}x\cdot\cot x\cdot\frac{dy}{dz} \qquad \text{...(ii)}$$

Put (i) and (ii) given differential equation, we get

$$\operatorname{cosec}^2x\cdot\frac{d^2y}{dz^2}-\operatorname{cosec}x\cdot\cot x\cdot\frac{dy}{dz}$$

$$+\cot x\cdot\operatorname{cosec}x\cdot\frac{dy}{dz}+4y\operatorname{cosec}^2x=0$$

$$\Rightarrow \frac{d^2y}{dz^2}+4y=0$$

Hence, value of k is 4.

57. (d) : Since, $x > x^2$ in $x \in (0, 1)$

$$\Rightarrow -x < -x^2 \Rightarrow \frac{-x^2}{2}>-x^2$$

$$\Rightarrow e^{-x^2/2}>e^{-x^2}$$

$\therefore \; I_4 > I_3$

Since, $\cos x$ is decreasing function in $x \in [0, 1]$

$$\therefore \; e^{-x^2/2}>e^{-x^2}\cos^2x$$

i.e, $I_4 > I_2$

Now, $-x^2 > -x$

Then $\frac{-x^2}{2} > -x$

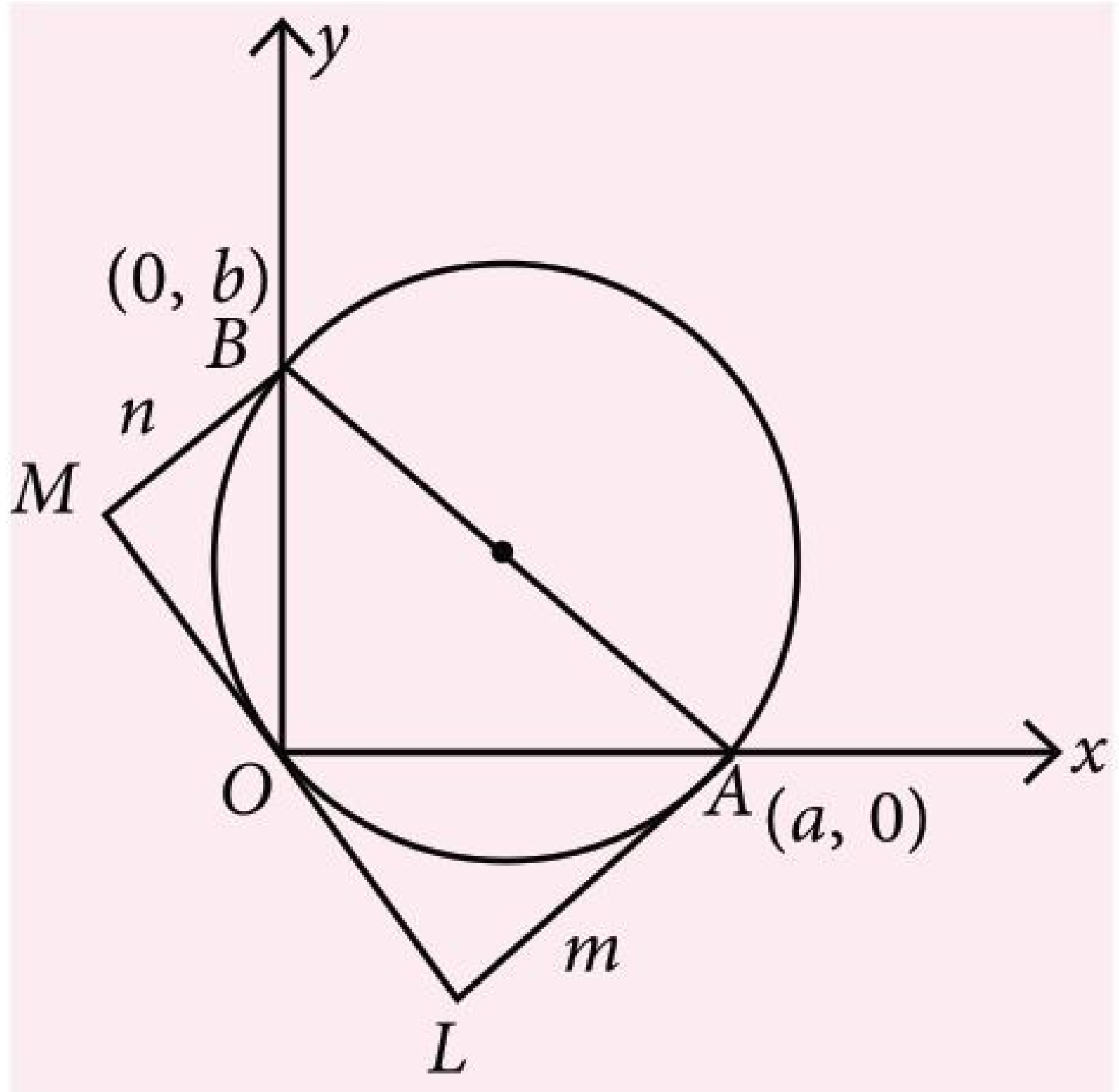
$\Rightarrow\ e^{-x^2/2} > e^{-x}\ \Rightarrow\ e^{-x^2/2} > e^{-x}\cos^2 x$

$\therefore\ I_4 > I_1$

Hence, I_4 is the greatest.

$I = I_4$

58. (b) : Length of AB



$$AB = \sqrt{a^2 + b^2}$$

Radius of circle is

$$\frac{1}{2}AB = \frac{1}{2}\sqrt{a^2 + b^2}$$

Equation of circle is

$x^2 + y^2 - ax - by = 0$

Equation of tangent through the origin is

$ax + by = 0$

From figure,

$$AL = m = \frac{a^2}{\sqrt{a^2 + b^2}} \text{ and } BM = n = \frac{b^2}{\sqrt{a^2 + b^2}}$$

$$\therefore\ m + n = \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} = \sqrt{a^2 + b^2} = AB$$

$= AB$

Hence, diameter of circle is $m + n$.

59. (a) : Equation of tangent is

$yt = x + at^2$

For point T, $y = 0$

$x = -at^2$

Equation of normal

$y + tx = 2at + at^3$

For point G, $y = 0$

$x = 2a + at^2$

So, $T \equiv (-at^2, 0)$ and $G \equiv (2a + at^2, 0)$

Radius of circle through point P, T and G is

$r = a(1 + t^2)$

60. (d) : Given parabola is $y^2 = 4x$

Equation of tangent is $y = mx + \frac{1}{m}$

Since the point $(-1, -6)$ lies on the tangent, Thus

$$-6 = -m + \frac{1}{m}$$

$\Rightarrow\ m^2 - 6m - 1 = 0$

$\Rightarrow\ m = 3 + \sqrt{10}, 3 - \sqrt{10}$

Let $m_1 = 3 + \sqrt{10},\ m_2 = 3 - \sqrt{10}$

$$\therefore\ \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{(3+\sqrt{10}) - (3-\sqrt{10})}{1 + (9 - 10)}\right|$$

$\Rightarrow\ \tan\theta = \infty\ \Rightarrow\ \theta = \frac{\pi}{2}$

61. (d)

62. (c) : Given, $f(x) = e^{\sin x} + e^{\cos x};\ x \in R$

$\Rightarrow\ f'(x) = e^{\sin x}\cos x + e^{\cos x}(-\sin x)$

For maximum value, put $f'(x) = 0$

$\Rightarrow\ e^{\sin x}\cos x + e^{\cos x}(-\sin x) = 0$

$\Rightarrow\ e^{\sin x}\cos x = e^{\cos x}\sin x$

Clearly, at $x = \frac{\pi}{4}$, the above equation holds.

So, $f(x)$ will be maximum at $x = \frac{\pi}{4}$

$$\therefore\ f\left(\frac{\pi}{4}\right) = e^{\sin\frac{\pi}{4}} + e^{\cos\frac{\pi}{4}} = 2e^{1/\sqrt{2}}$$

63. (b) : We have, $\log_{25}x^2 + (\log_5 x)^2 < 2$

$\Rightarrow\ \log_{5^2} x^2 + (\log_5 x)^2 < 2$

$\Rightarrow\ \frac{2}{2}\log_5 x + (\log_5 x)^2 < 2$ ...(i)

Let $\log_5 x = t$

$t + t^2 < 2\ \Rightarrow\ (t + 2)(t - 1) < 0\ \Rightarrow -2 < t < 1$

From (i), we get

$\Rightarrow\ -2 < \log_5 x < 1\ \Rightarrow\ 5^{-2} < x < 5^1$

$\Rightarrow\ x \in \left(\frac{1}{25}, 5\right)$

64. (d) : Given matrix $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$

Now, $\det\begin{bmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{bmatrix} = 0$

$\Rightarrow\ (2 - \lambda)(y - \lambda) - 3x = 0$

when $\lambda = 4$, we have

$-2(y - 4) - 3x = 0$

$\Rightarrow\ 3x + 2y = 8$ ...(i)

when $\lambda = 8$, we have

$-6(y - 8) - 3x = 0$

$\Rightarrow\ 3x + 6y = 48$

$\Rightarrow\ x + 2y = 16$ ...(ii)

Solving (i) and (ii), we get,

$x = -4,\ y = 10$

65. (b) : Given equation is,

$$x^2 - (a - 2)x - a - 1 = 0$$

or $x^2 - (a - 2)x - (a + 1) = 0$ ...(i)

Let α and β be the roots of (i)

$\therefore\ \alpha + \beta = a - 2$ and $\alpha\cdot\beta = -(a + 1)$

Now, $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow\ (a - 2)^2 = \alpha^2 + \beta^2 + 2 \times [-(a + 1)]$

$\Rightarrow\ \alpha^2 + \beta^2 = (a - 2)^2 + 2(a + 1)$

$= (a - 1)^2 + 5$

$\Rightarrow$ At $a = 1$, $\alpha^2 + \beta^2$ will be minimum.

66. (c) : Let (h, k) be the mid-point of chord drawn through the positive end of the minor axis of ellipse. Then, equation of chord is

$$\frac{hx}{a^2}+\frac{ky}{b^2}-1=\frac{h^2}{a^2}+\frac{k^2}{b^2}-1$$

$$\Rightarrow\ \frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

Since it pass through $(0, b)$. Therefore

$$\frac{k}{b}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

$\therefore$ Locus of mid point (h, k) is

$$\frac{k}{b}=\frac{x^2}{a^2}+\frac{y^2}{b^2},$$

which is an equation of ellipse.

67. (b, c) : Given equation $y - y_1 = m(x - x_1)$

Since m and x_1 are fixed. So, all different lines will have same slope m. Therefore all different lines are parallel. There x coordinate x_1 will also same. So all lines intersect the line $x = x_1$.

68. (b, d) : Since, the intersection of two equivalence relation on a set is again an equivalence relation.

But this is not true for union.

For example : Let A = {1, 2, 3}

Where R = {(1, 1), (2, 2), (1, 2), (2, 1)} and

S = {(2, 2), (2, 3), (3, 2), (3, 3)} be two equivalence relations

Then, R ∪ S = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

Since, clearly R ∪ S is reflexive and symmetric.

But for (1, 2), (2, 3) ∈ R ∪ S (1, 3) ∉ R ∪ S

$\therefore$ R ∪ S is not a transitive relation

Thus, R ∪ S is not an equivalence relation.

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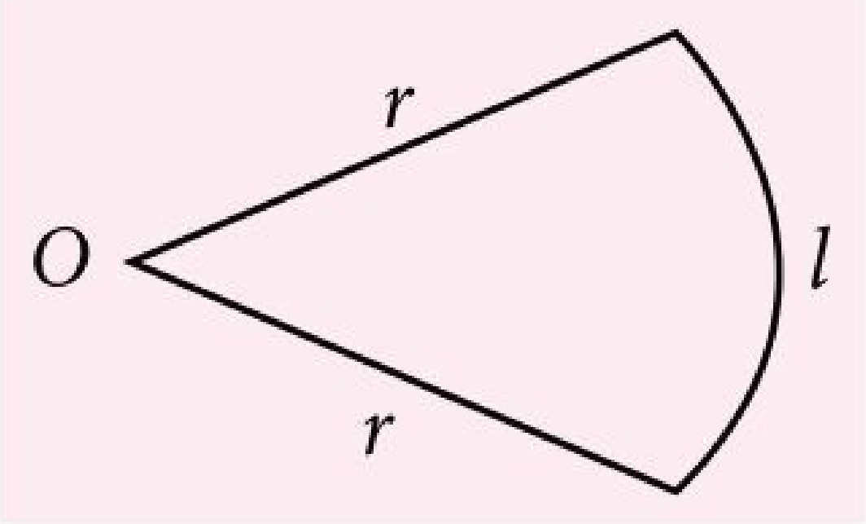
69. (c) : Let r is the radius of circle and l be the length of an arc of that circular sector.

$\therefore$ $20 = 2r + l$

$\Rightarrow$ $l = 20 - 2r$

Area, $A = \frac{1}{2}lr = \frac{1}{2}(20-2r)r$

For the maximum area, $\frac{dA}{dr} = 0$



i.e., $= \frac{1}{2}[20-4r] = 0$

$\Rightarrow$ $4r = 20$

$\Rightarrow$ $r = 5$

Radius must be 5 m if the area of the flower bed to be greatest.

70. (a, b, c) : Given $y = x + 5$

Comparing with $y = mx + c$, $m = 1$, $c = 5$.

Line $y = mx + c$ touches parabola $y^2 = 4ax$ then $c = \frac{a}{m}$.

For parabola $y^2 = 20x$ we have, $a = 5$ as $5 = \frac{5}{1}$

Hence, line $y = x + 5$ touches parabola $y^2 = 20x$

For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, c = \pm\sqrt{a^2m^2 + b^2}$

$\Rightarrow$ $c = \sqrt{4^2 + 3^2} = 5$ [As $a^2 = 4^2$ and $b^2 = 3^2$]

$\Rightarrow$ $5 = 5$

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

$c = \pm\sqrt{a^2m^2 - b^2}$ [As $a^2 = 29$ and $b^2 = 4$]

$\Rightarrow$ $c = \sqrt{29-4}$ $\Rightarrow$ $5 = 5$

And for circle of distance from the centre is equal to the radius of the circle then the line is a tangent.

Put $5 \neq \frac{5}{\sqrt{2}}$

$\therefore$ $y = x + 5$ does not touch the circle $x^2 + y^2 = 25$

71. (c) : Let $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$

$p(0) = a_0 = 1$

$p'(x) = a_1 + 2a_2x + 3a_3x^2 + ... + na_nxn^{-1}$

Hence, $p(x)$ may have negative real root.

72. (c) : Since h_1 is height when stone is released from balloon so it will take velocity of balloon.

$$S = ut + \frac{1}{2}gt^2 \Rightarrow -h_1 = 4v - \frac{1}{2} \times 32 \times 4^2$$

$\Rightarrow$ $h_1 = 256 - 4v$ ft

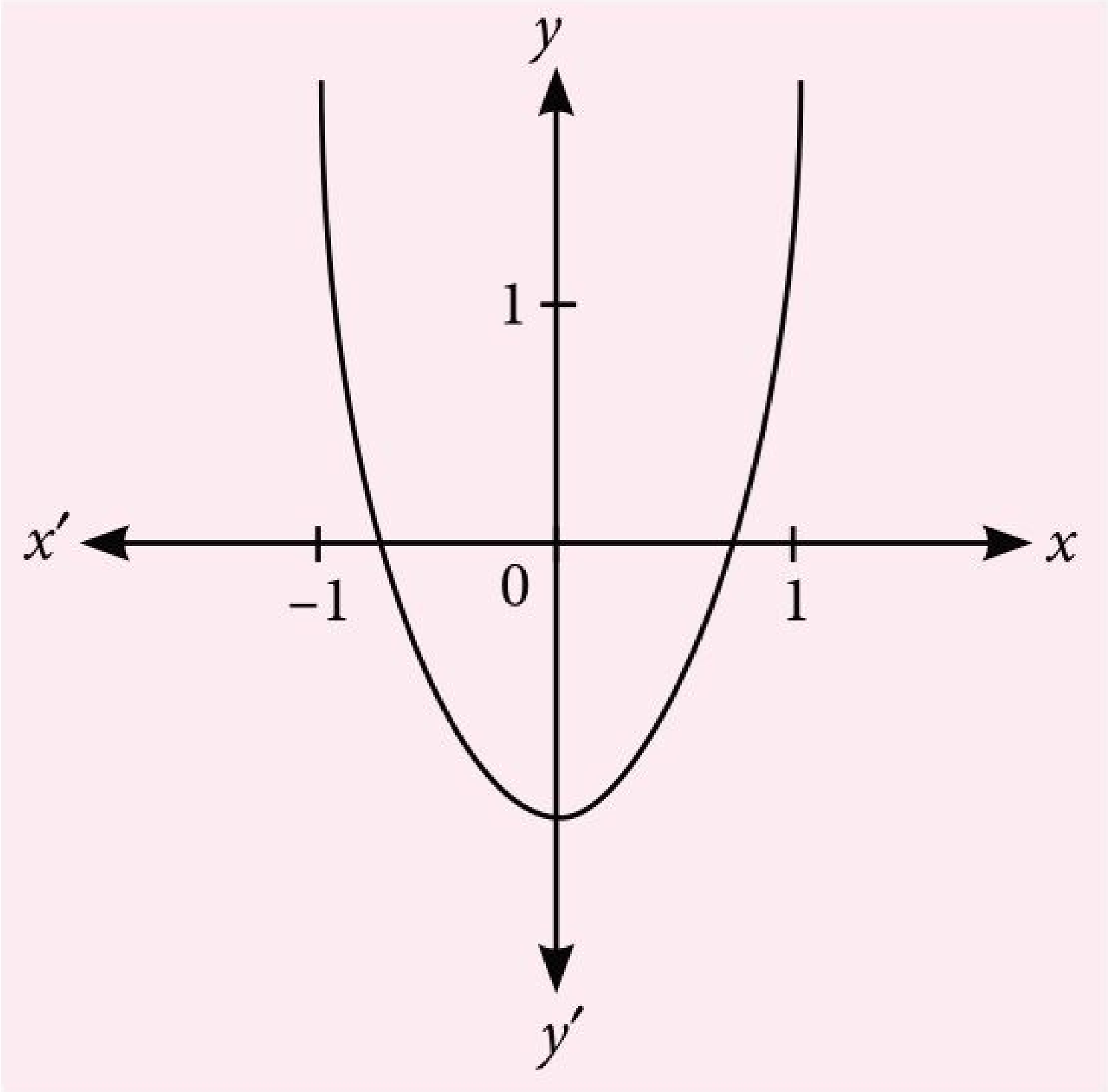
At the same time $t = 4$ sec, when balloon will travel with uniform velocity v. So, height $h_2 = 4v$ ft

$\therefore$ Total height achieved by balloon,

$h_1 + h_2 = 256 - 4v + 4v = 256$ ft

73. (a, c, d) : Graph of $f(x) = x^2 + x\sin x - \cos x$ will be quadratic in nature.

$f(0) = -1$



From the graph, we can see that $f(x) = 0$ has two roots in which one is positive root and other is negative root.

74. (a, d)

75. (b, d) : $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$

$= \sin\theta \cos\phi (\sin^2\theta \cos\phi) - \sin\theta \sin\phi (-\sin^2\theta \sin\phi)$
$+ \cos\theta [\cos\theta \sin\theta \cos^2\phi + \sin\theta \cos\theta \sin^2\phi]$

$= \sin^3\theta \cos^2\phi + \sin^3\theta \sin^2\phi + \cos\theta(\sin\theta \cos\theta)$

$= \sin^3\theta + \sin\theta \cos^2\theta = \sin\theta (\sin^2\theta + \cos^2\theta) = \sin\theta$

Clearly, Δ is independent of ϕ,

and $\frac{d\Delta}{d\theta} = \cos\theta \Rightarrow \left(\frac{d\Delta}{d\theta}\right)_{\theta = \frac{\pi}{2}} = \cos\frac{\pi}{2} = 0$

◆ ◆

COMIC CAPSULE

The Romans didn't find algebra very challenging, because X was always 10.

CBSE warm-up!

Term-II
Exam on
7th June

CLASS-XII

Practice Paper 2022

Time allowed : 2 Hours
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 3 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

2. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$.

3. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ with respect to x.

OR

Write the value of $\int \frac{2-3\sin x}{\cos^2 x}\,dx$.

4. Given $P(A) = 0.4$, $P(B) = 0.7$ and $P(B|A) = 0.6$. Find $P(A \cup B)$.

5. Let $\vec{a}$ and $\vec{b}$ are non-collinear. If $\vec{c} = (x-2)\vec{a}+\vec{b}$ and $\vec{d} = (2x+1)\vec{a}-\vec{b}$ are collinear, then find the value of x.

6. Find the integrating factor of the differential equation $(y+3x^2)\frac{dx}{dy} = x$.

SECTION - B

7. Evaluate : $\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

OR

Evaluate : $\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}}\,dx$

8. Solve : $\cos x\,(1 + \cos y)\,dx - \sin y\,(1 + \sin x)\,dy = 0$

9. Find the equation of a plane through the intersection of the planes $\vec{r}\cdot(\hat{i} + 3\hat{j} - \hat{k}) = 5$ and $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) = 3$ and passing through the point (2, 1, –2).

10. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}+7\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+2\hat{k}$ form the sides of a right-angled triangle.

SECTION - C

11. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest diseases are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian?

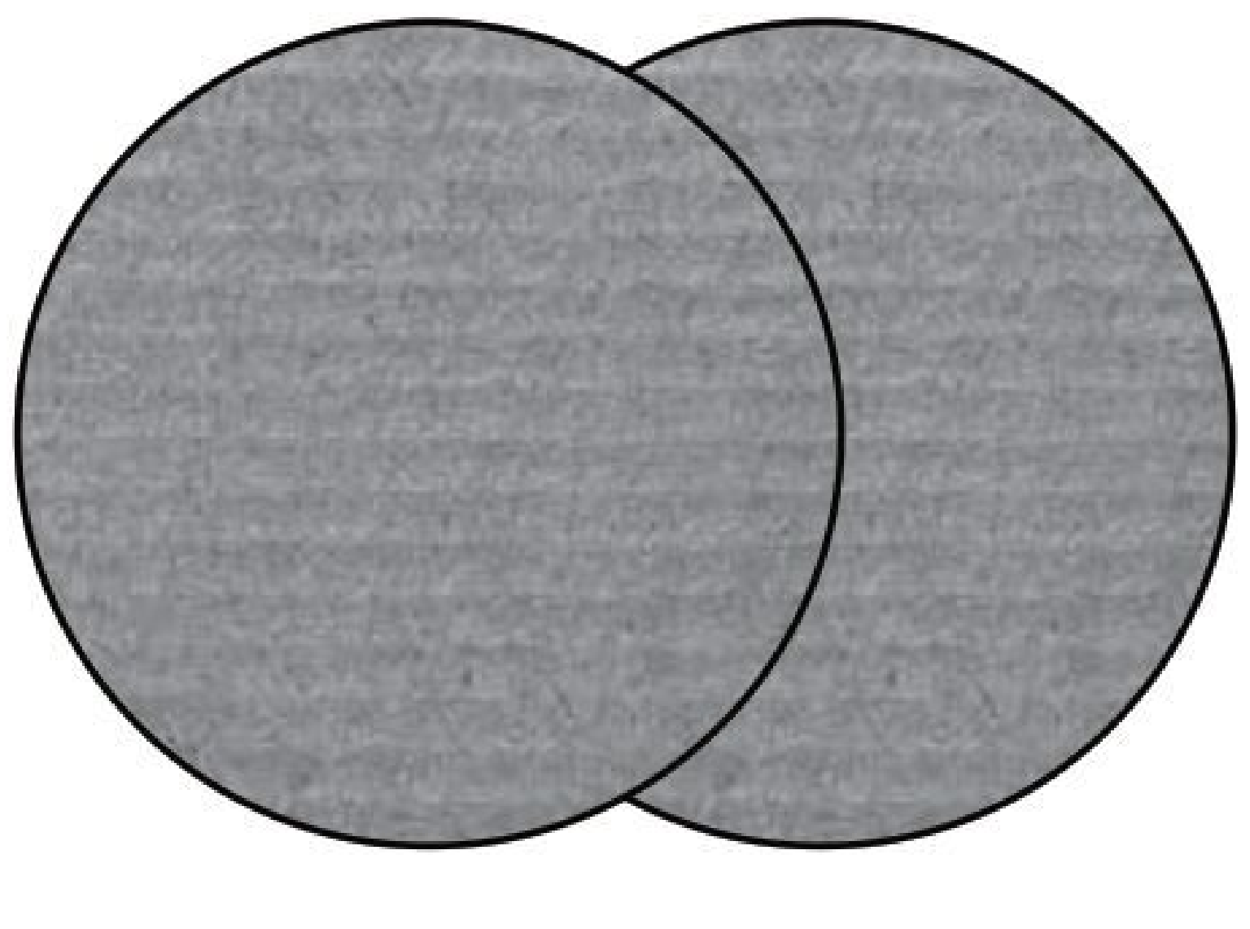
OR

A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant, 40%. Also, 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be of standard quality. Find the probability that it comes from the second plant.

12. Find the vector equation of the plane that contains the line $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and the point $(-1, 3, -4)$. Also find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained.

13. Evaluate : $\int_{-1}^{2}|x^3-3x^2+2x|\,dx$

14. Ajay cut two circular pieces of cardboard and placed one upon other as shown in figure. One of the circle represents the equation $(x-1)^2+y^2=1$, while other circle represents the equation $x^2+y^2=1$.



Based on the above information, answer the following questions.

(i) Draw the graph of given two curves.

(ii) Find the area of hidden portion of lower circle.

SOLUTIONS

1. Let A, B, C be respectively the events of solving problem by three students and $P(A)$, $P(B)$, $P(C)$ be their probability of solving the problem respectively.

$\therefore\ P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$ and $P(C)=\frac{1}{6}$

Required probability $=1-P(\bar{A}\cap\bar{B}\cap\bar{C})$

$=1-P(\bar{A})\,P(\bar{B})\,P(\bar{C})$

($\because$ A, B, C are independent $\therefore$ $\bar{A}$, $\bar{B}$, $\bar{C}$ are also independent)

$=1-[1-P(A)]\,[1-P(B)]\,[1-P(C)]$

$$=1-\frac{2}{3}\times\frac{3}{4}\times\frac{5}{6}=1-\frac{5}{12}=\frac{7}{12}$$

2. The given plane is $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$

$\Rightarrow\ \vec{n}=\hat{i}+2\hat{j}-5\hat{k}$

$\because$ D.r's of normal to this plane are 1, 2, –5.

So, the line has direction ratios proportional to 1, 2, –5.

$\therefore$ Eq. of line through (1, 2, 3) and $\perp$ to the plane is

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}+2\hat{j}-5\hat{k})$.

3.
$$\int\frac{\cos(x+a)}{\sin(x+b)}dx=\int\frac{\cos(x+b+a-b)}{\sin(x+b)}dx$$

$$=\int\frac{\cos(x+b)\cos(a-b)-\sin(x+b)\sin(a-b)}{\sin(x+b)}dx$$

$$=\cos(a-b)\int\frac{\cos(x+b)}{\sin(x+b)}dx-\sin(a-b)\int dx$$

$=\cos(a-b)\log\sin(x+b)-x\sin(a-b)+C$

OR

We have, $\int\frac{2-3\sin x}{\cos^2 x}dx=\int\left(\frac{2}{\cos^2 x}-\frac{3\sin x}{\cos^2 x}\right)dx$

$=\int(2\sec^2 x-3\sec x\tan x)dx=2\tan x-3\sec x+C$

4. We have, $P(A)=0.4$, $P(B)=0.7$ and $P(B\,|\,A)=0.6$

Now, $P(B\,|\,A)=\frac{P(A\cap B)}{P(A)}$

$\Rightarrow\ P(A\cap B)=P(B\,|\,A)\times P(A)$

$\therefore\ P(A\cap B)=0.6\times0.4=0.24$

Now, we have

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\ P(A\cup B)=0.4+0.7-0.24\ \Rightarrow P(A\cup B)=0.86$

5. We have, $\vec{c}=(x-2)\,\vec{a}+\vec{b}$, $\vec{d}=(2x+1)\,\vec{a}-\vec{b}$ are collinear, then $\vec{c}=m\vec{d}$

$\Rightarrow \quad (x-2)\vec{a}+\vec{b}=m\big((2x+1)\vec{a}-\vec{b}\big)$

$\Rightarrow \quad -m=1 \Rightarrow m=-1$

and $m(2x+1)=x-2 \Rightarrow -2x-1=x-2 \Rightarrow x=1/3$

6. We have, $(y+3x^2)\dfrac{dx}{dy}=x$

$$\Rightarrow \frac{y+3x^2}{x}=\frac{dy}{dx} \Rightarrow \frac{dy}{dx}-\frac{y}{x}=3x$$

This is a linear differential equation.

$$\therefore \quad \text{I.F.} = e^{-\int \frac{dx}{x}} = e^{-\log x} = e^{\log(x)^{-1}} = \frac{1}{x}$$

7. Let $I=\int e^{2x}\left(\dfrac{1-\sin 2x}{1-\cos 2x}\right)dx$

Put $2x=t \Rightarrow \quad dx=\dfrac{1}{2}dt$

$$\therefore \quad I=\frac{1}{2}\int e^t\left(\frac{1-\sin t}{1-\cos t}\right)dt=\frac{1}{2}\int e^t\left(\frac{1-2\sin\frac{t}{2}\cos\frac{t}{2}}{2\sin^2\frac{t}{2}}\right)dt$$

$$=\frac{1}{2}\int e^t\left(\frac{1}{2}\operatorname{cosec}^2\frac{t}{2}-\cot\frac{t}{2}\right)dt$$

$$=\frac{1}{2}\int e^t\left(-\cot\frac{t}{2}+\frac{1}{2}\operatorname{cosec}^2\frac{t}{2}\right)dt$$

[Compare with $\int e^x[f(x)+f'(x)]dx = e^x f(x)$

Take $f(t) = -\cot\dfrac{t}{2} \Rightarrow f'(t) = \dfrac{1}{2}\operatorname{cosec}^2\dfrac{t}{2}$]

$$\therefore \quad I=\frac{1}{2}e^t\left(-\cot\frac{t}{2}\right)+C=\frac{-1}{2}e^{2x}\cot x+C$$

OR

$$\text{Let } I=\int\frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx=\int\frac{\sin x-\cos x}{\sqrt{1+\sin 2x-1}}dx$$

$$=\int\frac{\sin x-\cos x}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x-1}}dx$$

$$=\int\frac{\sin x-\cos x}{\sqrt{(\sin x+\cos x)^2-1}}\cdot dx$$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore \quad I=\int\frac{-dt}{\sqrt{t^2-1}}=-\log|t+\sqrt{t^2-1}|+C$$

$$=-\log|\sin x+\cos x+\sqrt{\sin 2x}|+C$$

8. The given equation is $\cos x\,(1+\cos y)\,dx - \sin y\,(1+\sin x)\,dy = 0$

$$\Rightarrow \frac{\cos x}{1+\sin x}dx-\frac{\sin y}{1+\cos y}dy=0$$

$$\Rightarrow \int\frac{\cos x}{1+\sin x}dx+\int\frac{-\sin y}{1+\cos y}dy=0$$

[Integrating both sides]

$\Rightarrow \log|1+\sin x| + \log|1+\cos y| = \log C$

$\Rightarrow \log(|1+\sin x|\cdot|1+\cos y|) = \log C$

$\Rightarrow |1+\sin x|\,|1+\cos y| = C$

$\Rightarrow (1+\sin x)(1+\cos y) = C$

This is the required solution.

9. The equation of a plane through the intersection of $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$ is

$[\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})-5]+\lambda\,[\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-3]=0$

$$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(-1+\lambda)\hat{k}]-(5+3\lambda)=0 \qquad \text{...(i)}$$

If plane in (i) passes through (2, 1, – 2), then the vector $2\hat{i}+\hat{j}-2\hat{k}$ should satisfy it.

$$\therefore \quad (2\hat{i}+\hat{j}-2\hat{k})\cdot[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(-1+\lambda)\hat{k}]-(5+3\lambda)=0$$

$\Rightarrow 2(1+2\lambda)+1(3-\lambda)-2(-1+\lambda)-(5+3\lambda)=0$

$\Rightarrow -2\lambda+2=0 \Rightarrow \lambda=1$

Putting $\lambda = 1$ in (i), we get the required equation of the plane as $\vec{r}\cdot(3\hat{i}+2\hat{j}+0\hat{k})=8$.

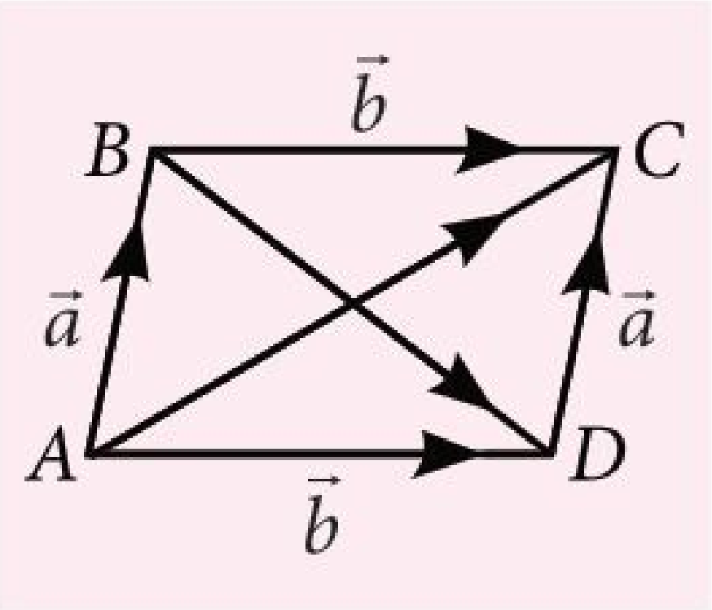
10. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$

Then diagonal $\overrightarrow{AC}$ of the parallelogram is

$\vec{p}=\vec{a}+\vec{b}$

$=\hat{i}+2\hat{j}+3\hat{k}+2\hat{i}+4\hat{j}-5\hat{k}$

$=3\hat{i}+6\hat{j}-2\hat{k}$



Therefore unit vector parallel to it is

$$\frac{\vec{p}}{|\vec{p}|}=\frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}}=\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$$

Now, diagonal $\overrightarrow{BD}$ of the parallelogram is

$\vec{p}\,'=\vec{b}-\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}=\hat{i}+2\hat{j}-8\hat{k}$

Therefore unit vector parallel to it is

$$\frac{\vec{p}\,'}{|\vec{p}\,'|}=\frac{\hat{i}+2\hat{j}-8\hat{k}}{\sqrt{1+4+64}}=\frac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$$

OR

Let $A(2\hat{i}-\hat{j}+\hat{k}), B(3\hat{i}+7\hat{j}+\hat{k})$ and $C(5\hat{i}+6\hat{j}+2\hat{k})$

Then, $\overrightarrow{AB}=(3-2)\hat{i}+(7+1)\hat{j}+(1-1)\hat{k}=\hat{i}+8\hat{j}$

$\overrightarrow{AC}=(5-2)\hat{i}+(6+1)\hat{j}+(2-1)\hat{k}=3\hat{i}+7\hat{j}+\hat{k}$

$\overrightarrow{BC}=(5-3)\hat{i}+(6-7)\hat{j}+(2-1)\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

Now, angle (θ) between $\overrightarrow{AC}$ and $\overrightarrow{BC}$ is given by

$$\Rightarrow \cos\theta=\frac{\overrightarrow{AC}\cdot\overrightarrow{BC}}{|\overrightarrow{AC}||\overrightarrow{BC}|}=\frac{6-7+1}{\sqrt{9+49+1}\sqrt{4+1+1}}$$

$\Rightarrow \cos\theta = 0 \Rightarrow AC \perp BC$

So, A, B, C are the vertices of right angled triangle.

11. Let A, B, C and E are respectively the events that a person is smoker and non-vegetarian, smoker and vegetarian, non-smoker and vegetarian, and the selected person is suffering from the disease.

Here, $n(A) = 160$, $n(B) = 100$,

$n(C) = 400 - (160 + 100) = 140$.

Also, $P(A)=\frac{160}{400}, P(B)=\frac{100}{400}, P(C)=\frac{140}{400}$

and $P(E/A)=\frac{35}{100}, P(E/B)=\frac{20}{100}, P(E/C)=\frac{10}{100}$

∴ Required probability

$$=P(A/E)=\frac{P(A)\cdot P(E/A)}{P(A)\cdot P(E/A)+P(B)\cdot P(E/B)+P(C)\cdot P(E/C)}$$

$$=\frac{\frac{160}{400}\times\frac{35}{100}}{\frac{160}{400}\times\frac{35}{100}+\frac{100}{400}\times\frac{20}{100}+\frac{140}{400}\times\frac{10}{100}}$$

$$=\frac{5600}{5600+2000+1400}=\frac{5600}{9000}=\frac{28}{45}$$

OR

Let E_1 and E_2 be the events of choosing a bicycle from the first plant and the second plant respectively.

Let E be the event of choosing a bicycle of standard quality. Then, $P(E_1)=\frac{60}{100}=\frac{3}{5}$, and $P(E_2)=\frac{40}{100}=\frac{2}{5}$.

$P(E|E_1)$ = probability of choosing a bicycle of standard quality, given that it is produced by the first plant = 80/100 = 4/5.

$P(E|E_2)$ = probability of choosing a bicycle of standard quality, given that it is produced by the second plant = 90/100 = 9/10.

∴ Required probability

= $P(E_2|E)$ = probability of choosing a bicycle from the second plant, given that it is of standard quality

$$=\frac{P(E_2)\cdot P(E\,|\,E_2)}{P(E_1)\cdot P(E\,|\,E_1)+P(E_2)\cdot P(E\,|\,E_2)}$$ [By Bayes' theorem]

$$=\frac{\left(\frac{2}{5}\times\frac{9}{10}\right)}{\left(\frac{3}{5}\times\frac{4}{5}\right)+\left(\frac{2}{5}\times\frac{9}{10}\right)}=\frac{3}{7}.$$

12. Given line is $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$

The line passes through point having position vector $\vec{a}=\hat{i}+\hat{j}$ and parallel to vector $\vec{b}_1=\hat{i}+2\hat{j}-\hat{k}$.

The plane passes through point (–1, 3, –4).

∴ Plane is also parallel to vector

$\vec{b}_2=(-1-1)\hat{i}+(3-2)\hat{j}+(-4+1)\hat{k}$ *i.e.*, $\vec{b}_2=-2\hat{i}+\hat{j}-3\hat{k}$

i.e., $\vec{b}_2=-2\hat{i}+\hat{j}-3\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\vec{n}=\begin{bmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-1\\-2&1&-3\end{bmatrix}$$

$$=\hat{i}(-6+1)-\hat{j}(-3-2)+\hat{k}(1+4)=-5\hat{i}+5\hat{j}+5\hat{k}$$

Now, plane passes through $\vec{a}=\hat{i}+\hat{j}$

and normal to vector $\vec{n}=-5\hat{i}+5\hat{j}+5\hat{k}$

Hence the equation of required plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0 \;\Rightarrow\; \vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

$\Rightarrow\; \vec{r}\cdot(-5\hat{i}+5\hat{j}+5\hat{k})=(\hat{i}+\hat{j})\cdot(-5\hat{i}+5\hat{j}+5\hat{k})$

$\Rightarrow\; \vec{r}\cdot(-5\hat{i}+5\hat{j}+5\hat{k})=0 \Rightarrow -5x+5y+5z=0$

∴ Length of perpendicular from (2, 1, 4) to plane

$$=\frac{(-5)(2)+5(1)+5(4)}{\sqrt{25+25+25}}=\frac{-10+5+20}{\sqrt{75}}=\frac{15}{\sqrt{75}}=\frac{15}{5\sqrt{3}}$$

$=\sqrt{3}$ units

13. The given definite integral $=\int_{-1}^{2}|x(x-1)(x-2)|\,dx$

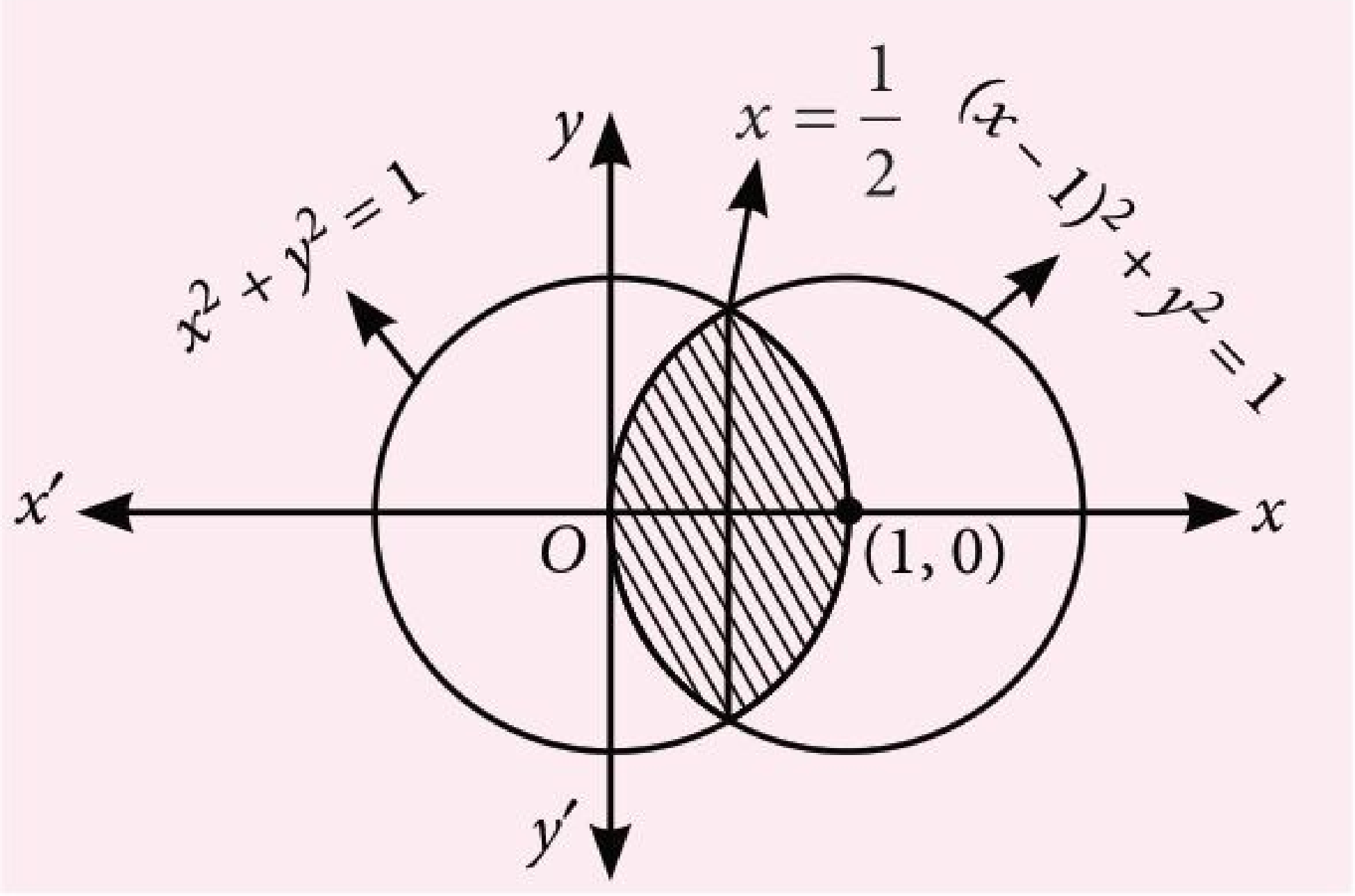
$$=\int_{-1}^{0}|x(x-1)(x-2)|\,dx+\int_{0}^{1}|x(x-1)(x-2)|\,dx+\int_{1}^{2}|x(x-1)(x-2)|\,dx$$

$$=-\int_{-1}^{0}(x^3-3x^2+2x)dx+\int_{0}^{1}(x^3-3x^2+2x)dx-\int_{1}^{2}(x^3-3x^2+2x)dx$$

$$=-\left[\frac{x^4}{4}-x^3+x^2\right]_{-1}^{0}+\left[\frac{x^4}{4}-x^3+x^2\right]_{0}^{1}-\left[\frac{x^4}{4}-x^3+x^2\right]_{1}^{2}$$

$$=\frac{9}{4}+\frac{1}{4}+\frac{1}{4}=\frac{11}{4}$$

14. (i)



(ii) Required area

$$=2\left[\int_{0}^{1/2}\sqrt{1-(x-1)^2}\,dx+\int_{1/2}^{1}\sqrt{1-x^2}\,dx\right]$$

$$=2\left[\frac{\pi}{6}-\frac{\sqrt{3}}{8}+\frac{\pi}{6}-\frac{\sqrt{3}}{8}\right]$$

$$=2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)\text{sq. units}$$

MONTHLY TEST DRIVE

Practice Paper

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If the value of $\prod_{k=1}^{50}\begin{bmatrix}1 & 2k-1\\ 0 & 1\end{bmatrix}$ is equal to $\begin{bmatrix}1 & r\\ 0 & 1\end{bmatrix}$, then r is equal to

(a) 62500 (b) 2500 (c) 1250 (d) 12500

2. The bag 'A' contains 5 white and 3 black balls while the bag 'B' contains 4 white and 7 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is white?

(a) $\frac{87}{176}$ (b) $\frac{81}{172}$ (c) $\frac{82}{179}$ (d) $\frac{87}{179}$

3. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+\hat{j}, \vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then $\lambda+\mu$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

4. If $f(x)=\frac{2-\sqrt{x+4}}{\sin(2x)}$, $x \neq 0$ is continuous function at $x = 0$, then the value of $f(0)$ is

(a) $-1/8$ (b) $1/2$ (c) 1 (d) 0

5. If $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$, $y(1)=\frac{\pi}{2}$, then $y\left(\frac{1}{2}\right)=$

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/12$

6. $\int\sqrt{1+\sec x}\,dx=$

(a) $\cos^{-1}\sqrt{\cos x}+c$ (b) $\sin^{-1}\sqrt{\cos x}+c$

(c) $2\cos^{-1}\sqrt{\cos x}+c$ (d) $2\sin^{-1}\sqrt{\cos x}+c$

One or More than One Option(s) Correct Type

7. Solutions of the differential equation

$$\left(\frac{dy}{dx}\right)^2-\frac{dy}{dx}(e^x+e^{-x})+1=0 \text{ are given by :}$$

(a) $y+e^{-x}=C$ (b) $y-e^{-x}=C$

(c) $y+e^{x}=C$ (d) $y-e^{x}=C$

8. If A and B are two symmetric matrices of the same order, then

(a) $A+B$ is symmetric (b) $A-B$ is symmetric

(c) AB is symmetric (d) BAB is symmetric

9. If the curves $y=2(x-a)^2$ and $y=e^{2x}$ touch each other, then 'a' is less than :

(a) -1 (b) 0 (c) 1 (d) 2

10. Let the parabolas $y=x^2+ax+b$ and $y=x(c-x)$ touch each other at point (1,0). Then

(a) $a=-3$ (b) $b=2$ (c) $c=2$ (d) $b+c=3$

11. If 'A' denotes area of the region bounded by $y=\sin^{-1}(x^2)$, $y=0$, $x=1$ and $x=-1$, then :

(a) $A=2\int_0^{\pi/2}(1-\sin^{-1}x^2)dx$

(b) $A=2\int_0^{\pi/2}(1-\sqrt{\sin x})dx$

(c) $0<A<\frac{\pi}{2}$ (d) $\frac{\pi}{2}<A<\pi$

12. The function $f(x)=\begin{cases}|x-3|, & x\geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, & x<1\end{cases}$ is

(a) continuous at $x=1$

(b) differentiable at $x=1$

(c) continuous at $x=3$

(d) differentiable at $x=3$

Comprehension Type

Paragraph for Q. No. 13 to 15

Let $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors and let the equations $\vec{a}' = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{b}' = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{c}' = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ are reciprocal system of vectors, $\vec{a}, \vec{b}, \vec{c}$.

13. The expression $\vec{a}\times\vec{a}' + \vec{b}\times\vec{b}' + \vec{c}\times\vec{c}'$ is.

(a) a unit vector
(b) a null vector
(c) equal to $\dfrac{\vec{a}^2+\vec{b}^2+\vec{c}^2}{\vec{a}'^2+\vec{b}'^2+\vec{c}'^2}$
(d) none of these

14. The value of the expression $\vec{a}'\times\vec{b}' + \vec{b}'\times\vec{c}' + \vec{c}'\times\vec{a}'$ is:

(a) $\dfrac{\vec{a}+\vec{b}-\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$
(b) $\dfrac{\vec{a}-\vec{b}+\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$
(c) $\dfrac{-\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$
(d) $\dfrac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

15. If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = \hat{i}-\hat{j}-2\hat{k}$, $\vec{c} = -\hat{i}+2\hat{j}+2\hat{k}$, then $\vec{c}'\times\vec{a}'$ equals :

(a) $\dfrac{\hat{i}+\hat{j}-2\hat{k}}{3}$
(b) $\dfrac{\hat{i}-\hat{j}-2\hat{k}}{3}$
(c) $\dfrac{\hat{i}+\hat{j}-2\hat{k}}{3}$
(d) $\dfrac{-\hat{i}+\hat{j}-2\hat{k}}{3}$

Matrix Match Type

16. Match the following.

	Column I		Column II
(P)	Let $\lvert A\rvert = \lvert a_{ij}\rvert_{3\times3} \neq 0$. Each element a_{ij} is multiplied by k^{i-j}. Let $\lvert B\rvert$ the resulting determinant, where $k_1\lvert A\rvert + k_2\lvert B\rvert = 0$. Then $k_1 + k_2 =$	(1)	0
(Q)	The maximum value of a third order determinant each of its entries are ±1 equals	(2)	4
(R)	If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma =$	(3)	1
(S)	$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax + B$ where A and B are determinants of order 3. Then $A + 2B =$	(4)	2

	P	Q	R	S
(a)	2	3	1	2
(b)	1	2	3	1
(c)	2	1	3	4
(d)	4	1	2	1

Numerical Answer Type

17. Let $f(x)$ be a cubic polynomial such that $\lim\limits_{x\to0}(1+f(x))^{1/x} = e^2$. If $x = -1$ is point of inflection of $f(x)$ and $f(1) = 3$, then $\left|\dfrac{f(1)}{f(-1)}\right| =$ ______.

18. If the shortest distance between the lines $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda_1(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r} = (2\hat{i}+4\hat{j}+5\hat{k}) + \lambda_2(3\hat{i}+4\hat{j}+5\hat{k})$ is x, then $\cos^{-1}(\cos\sqrt{6}x)$ is equal to ______.

19. The number of triplets satisfying $\sin^{-1}x + \cos^{-1}y + \sin^{-1}z = 2\pi$, is ______.

20. If a, b and c are three numbers (not necessarily different) chosen randomly and with replacement from the set {1, 2, 3, 4, 5}, the probability that $(ab + c)$ is even, is _______.

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No. of questions correct ......
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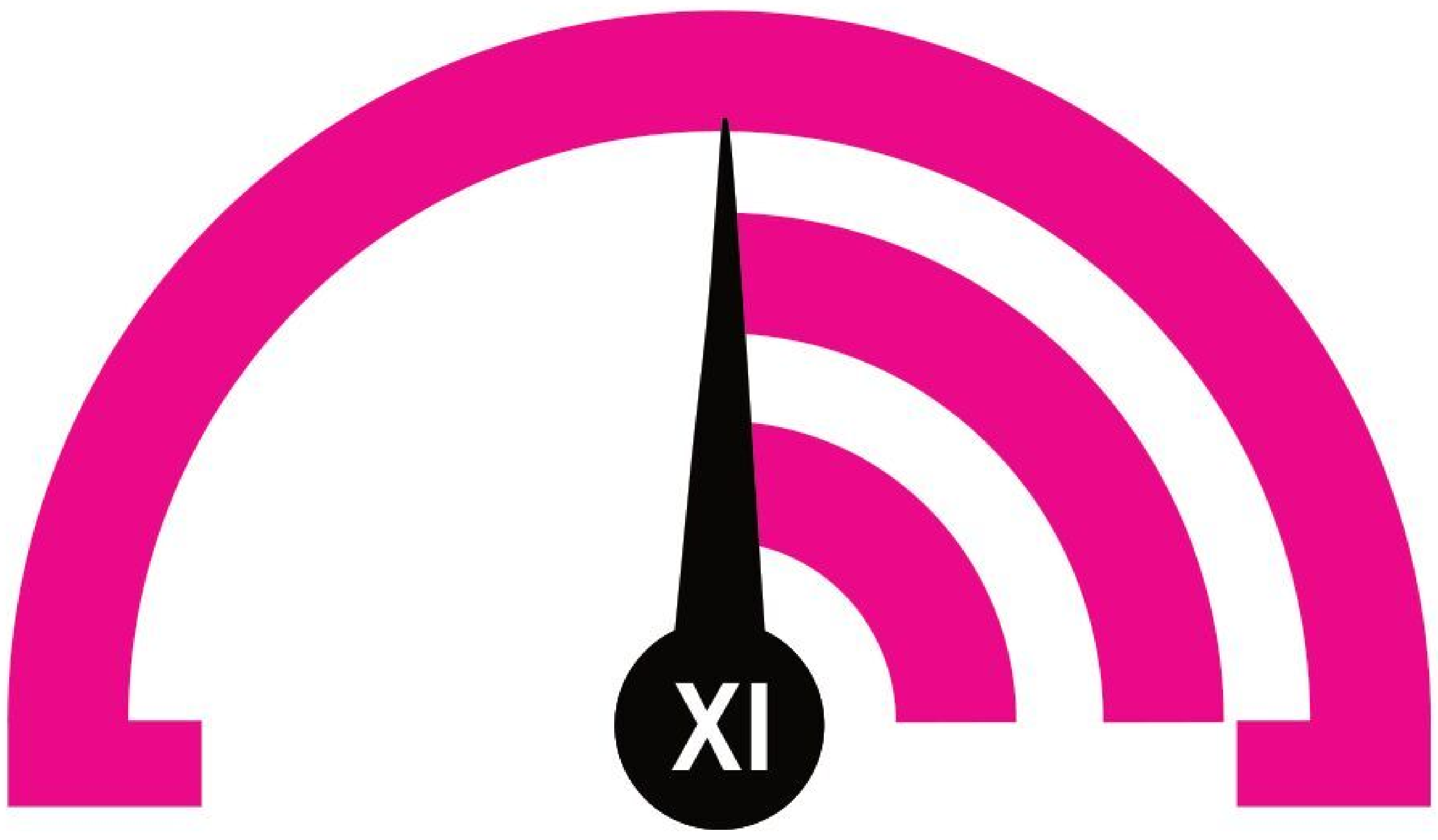
Practice Paper



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Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $e^{f(x)} = \frac{10+x}{10-x}$, $x \in (-10, 10)$ and $f(x) = kf\left(\frac{200x}{100+x^2}\right)$, then k is equal to

(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

2. Tangent is drawn at any point (p, q) on the parabola $y^2 = 4ax$. Tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$, such that the chords of contact pass through a fixed point (r, s), then p, q, r, s hold the relation

(a) $r^2q = 4p^2s$ (b) $rq^2 = 4ps^2$

(c) $rq^2 = -4ps^2$ (d) $r^2q = -4p^2s$

3. How many signals can be made by 5 flags from 8 flags of different colours?

(a) ${}^8C_5$ (b) ${}^8C_5 \times 5!$

(c) 5^8 (d) 8^5

4. If $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$ and $A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{x : x^2 - 3x + 2 = 0\}$, then $(A \cap B)' =$

(a) $\{1, 3\}$ (b) $\{1, 2, 3\}$

(c) $\{0, 1, 3\}$ (d) $\{0, 1, 2, 3\}$

5. If $n > 0$ is an odd integer, and $x = \left(\sqrt{2}+1\right)^n$ and $f = x - [x]$, then $\frac{1-f^2}{f}$ is

(a) an irrational number

(b) a non integer rational number

(c) an odd number

(d) an even integer

6. $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}+\frac{3}{n^2}+\frac{5}{n^2}+.....+\frac{2n+1}{n^2}\right]$ is equal to

(a) $\frac{1}{2}$ (b) 1 (c) $-\frac{1}{2}$ (d) -1

One or More than One Option(s) Correct Type

7. Which one is not periodic function?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos x / \cos\sqrt{2}x$

8. If a number of ellipse be described having the same major axis $2a$ but a variable minor axis, then the tangents at the end of their latus rectum pass through fixed points which can be

(a) $(0, a)$ (b) $(0, 0)$

(c) $(0, -a)$ (d) (a, a)

9. $\sin x$, $\sin 2x$, $\sin 3x$ are in A.P., if

(a) $x = \frac{n\pi}{2}$, $n \in I$ (b) $x = n\pi$, $n \in I$

(c) $x = 2n\pi$, $n \in I$ (d) $x = (2n+1)\pi, n \in I$

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10. If x, y, z are real numbers such that $x + y + z = 5$, $xy + yz + zx = 8$, then
(a) x can be 2 (b) y can be 1
(c) z can be 3/2 (d) z can be 5/2

11. If $[x^2] + x = a$ has solution(s), $a \in N$, then 'a' may be (where $[\cdot]$ denotes greatest integer function)
(a) 2 (b) 12 (c) 20 (d) 24

12. The equation of a circle of radius 1 touching the circles $x^2 + y^2 - 2|x| = 0$ is
(a) $x^2+y^2+2\sqrt{3}x-2=0$
(b) $x^2+y^2-2\sqrt{3}y+2=0$
(c) $x^2+y^2+2\sqrt{3}y+2=0$
(d) $x^2+y^2+2\sqrt{3}x+2=0$

Comprehension Type

Paragraph for Q. No. 13 to 15

In a survey of 950 families in a village, the following distribution of number of children was obtained.

Number of children	0-2	2-4	4-6	6-8	8-10	10-12
Number of families	272	328	205	120	15	10

13. Find the mean.
(a) 3.545 (b) 3.543 (c) 3.443 (d) 3.343

14. Find the S.D.
(a) 1.23 (b) 2.23 (c) 4.23 (d) 3.23

15. Find the median.
(a) 2.238 (b) 4.238 (c) 3.238 (d) 5.238

Matrix Match Type

16. Match the following.

	Column I		**Column II**
(P)	If the centroid of the triangle is origin and two of its vertices are (3, –5, 7)and (–1, 7, –6), then the third vertex is	(1)	the vertices of a parallelogram
(Q)	If the mid-points of the sides of a triangle are (1, 2, –3), (3, 0, 1) and (–1, 1, –4), then the centroid of the triangle is	(2)	(–2, –2, –1)
(R)	The points $A(3, -1, -1)$, $B(5, -4, 0)$, $C(2, 3, -2)$ and $D(0, 6, -3)$ are	(3)	(1, 1, –2)
(S)	Points $A(2, -1, 3)$, $B(3, -4, 5)$ and $C(6, -13, 11)$ are	(4)	collinear

	P	**Q**	**R**	**S**
(a)	2	3	1	4
(b)	3	2	4	1
(c)	4	1	2	3
(d)	1	4	2	3

Numerical Answer Type

17. If $(1 + i)^5 + (1 - i)^8 = a + ib$, then $a + b =$____.

18. The sum of three numbers in G.P. is 42. If each of the extremes be multiplied by 4 and the mean by 5, then the products are in A.P. The least of the original numbers is ____.

19. If 10 ten rupee coins, 5 five rupee coins are to be placed in a line such that the probability that the extreme coins are five rupee coins is p, then the value of $21p$ is ____.

20. The line $x + y = k$ intersects the x-axis and the y-axis at P and Q respectively. Let L be any point on OQ and LM be the perpendicular on the line PQ. If area of ΔLMP is $\frac{3}{8}$th of the area of ΔPQO and the ratio $PM : MQ = \lambda : 1$, then $\frac{2\lambda}{3}$ is____.

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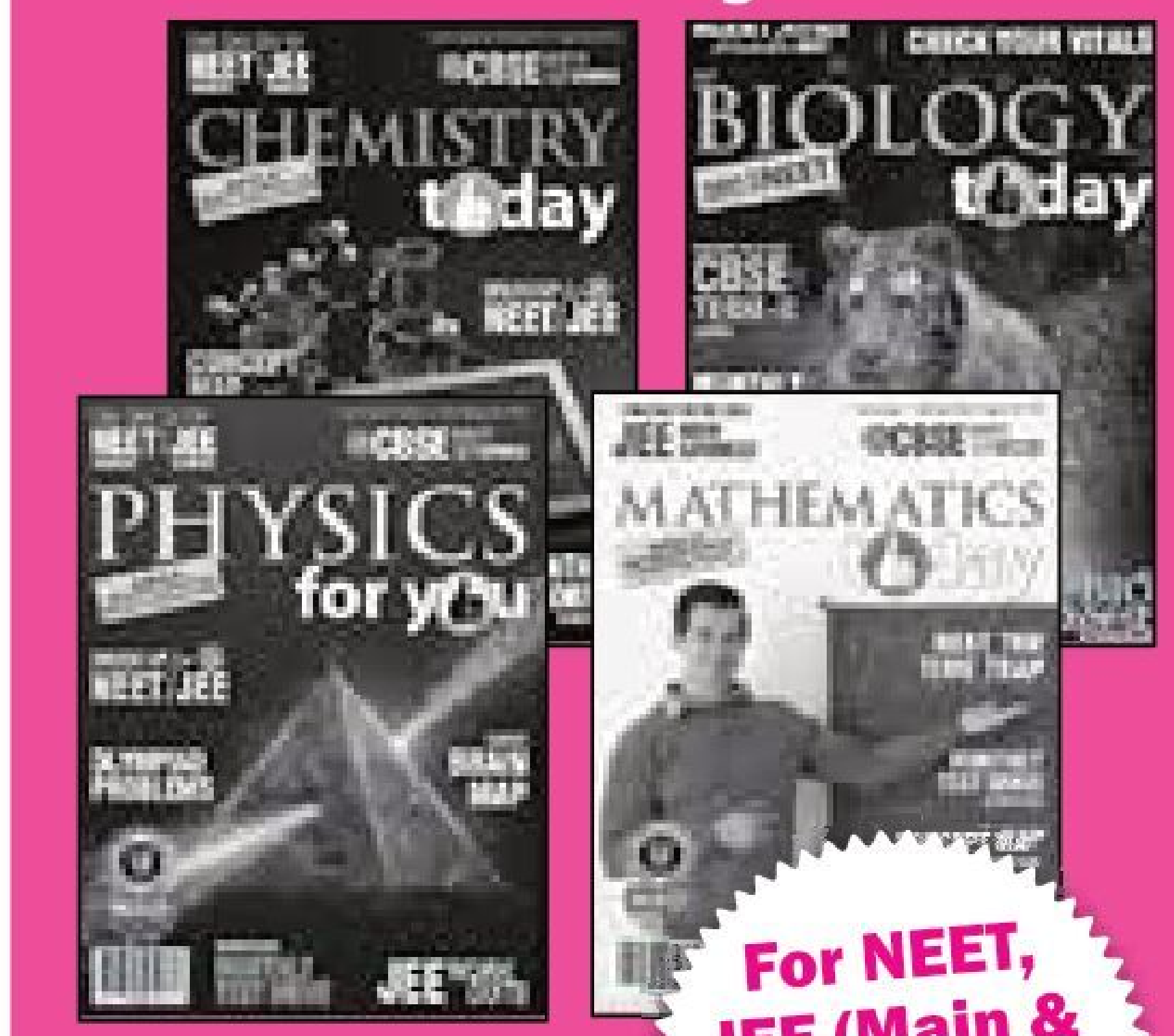

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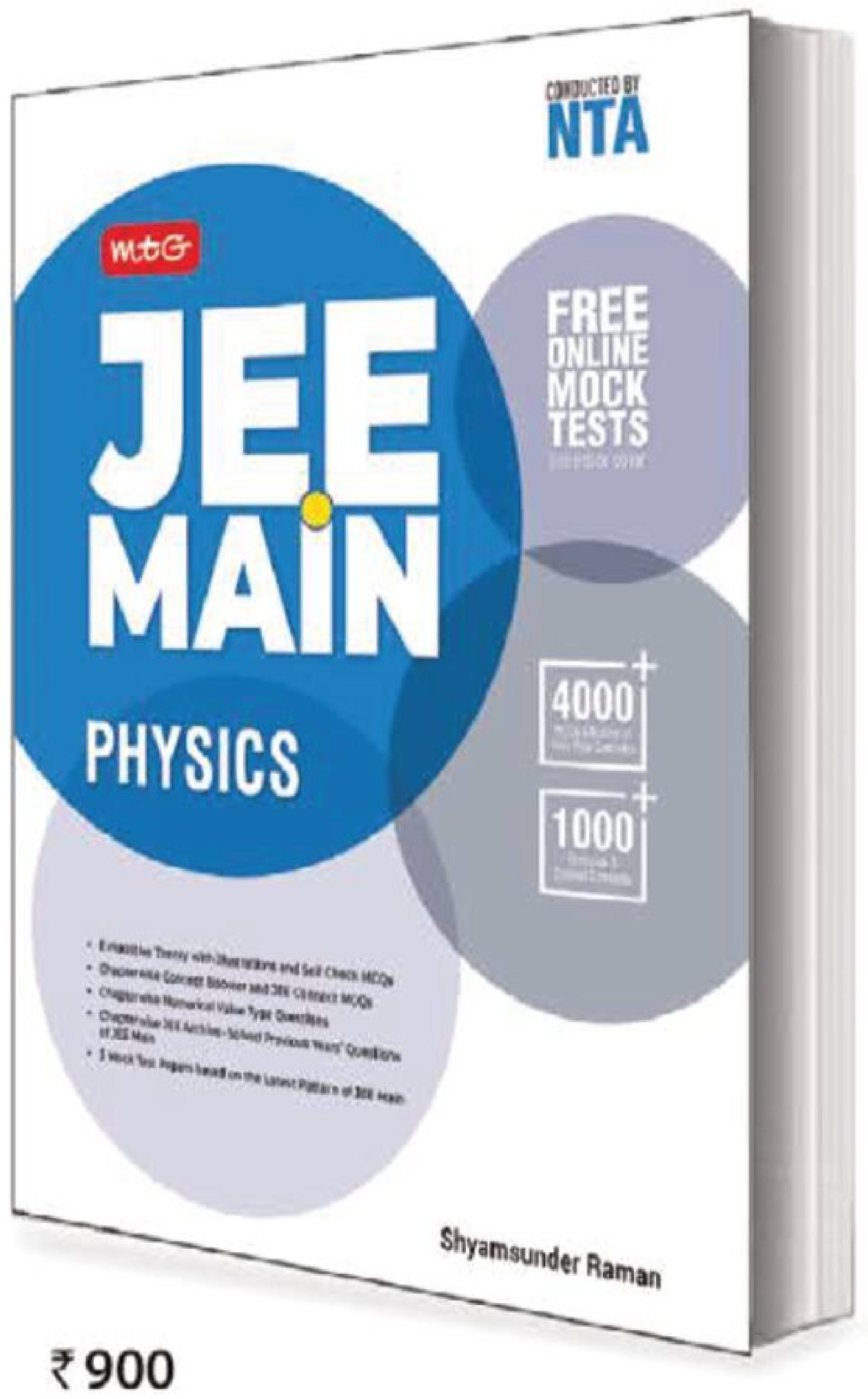
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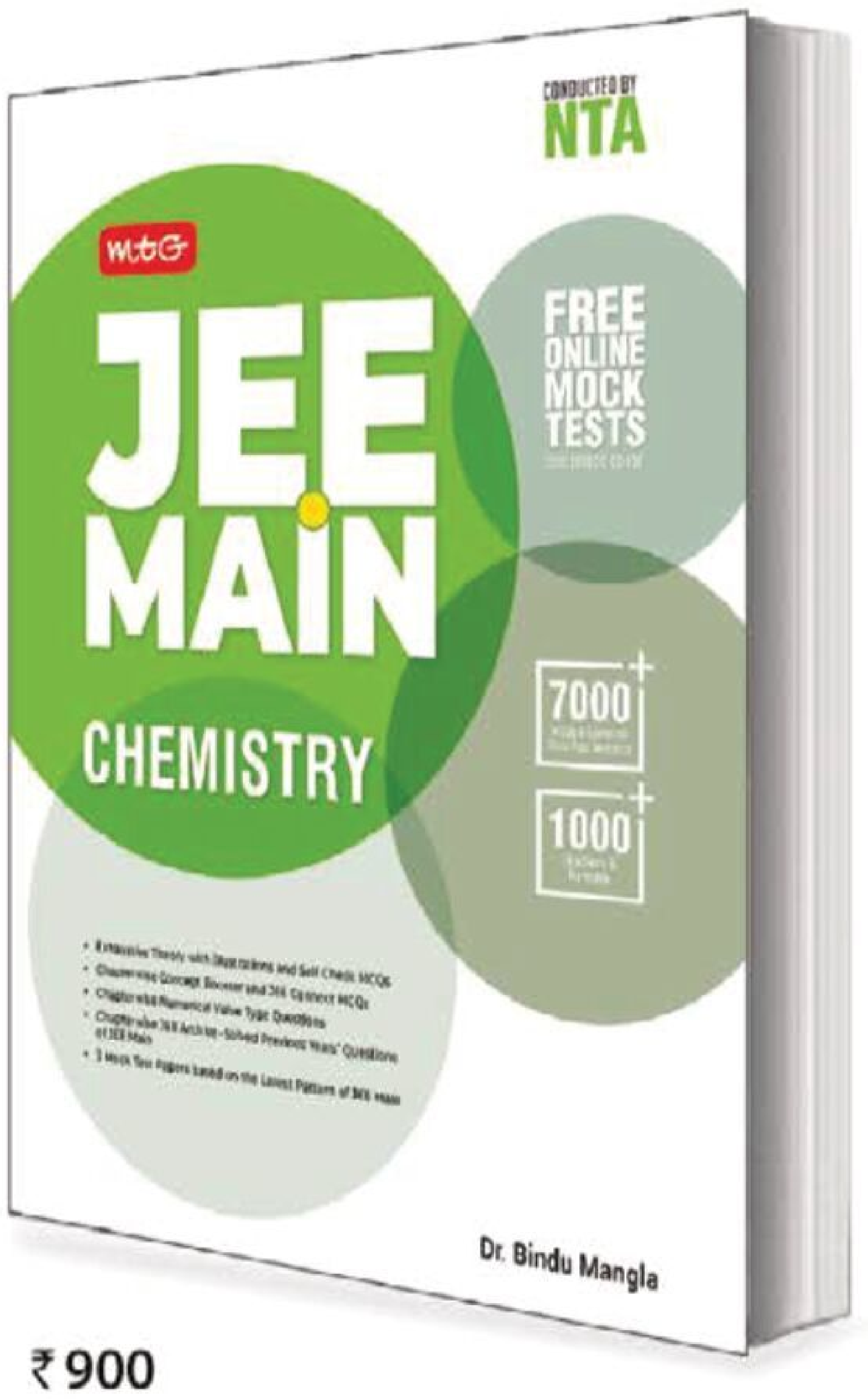
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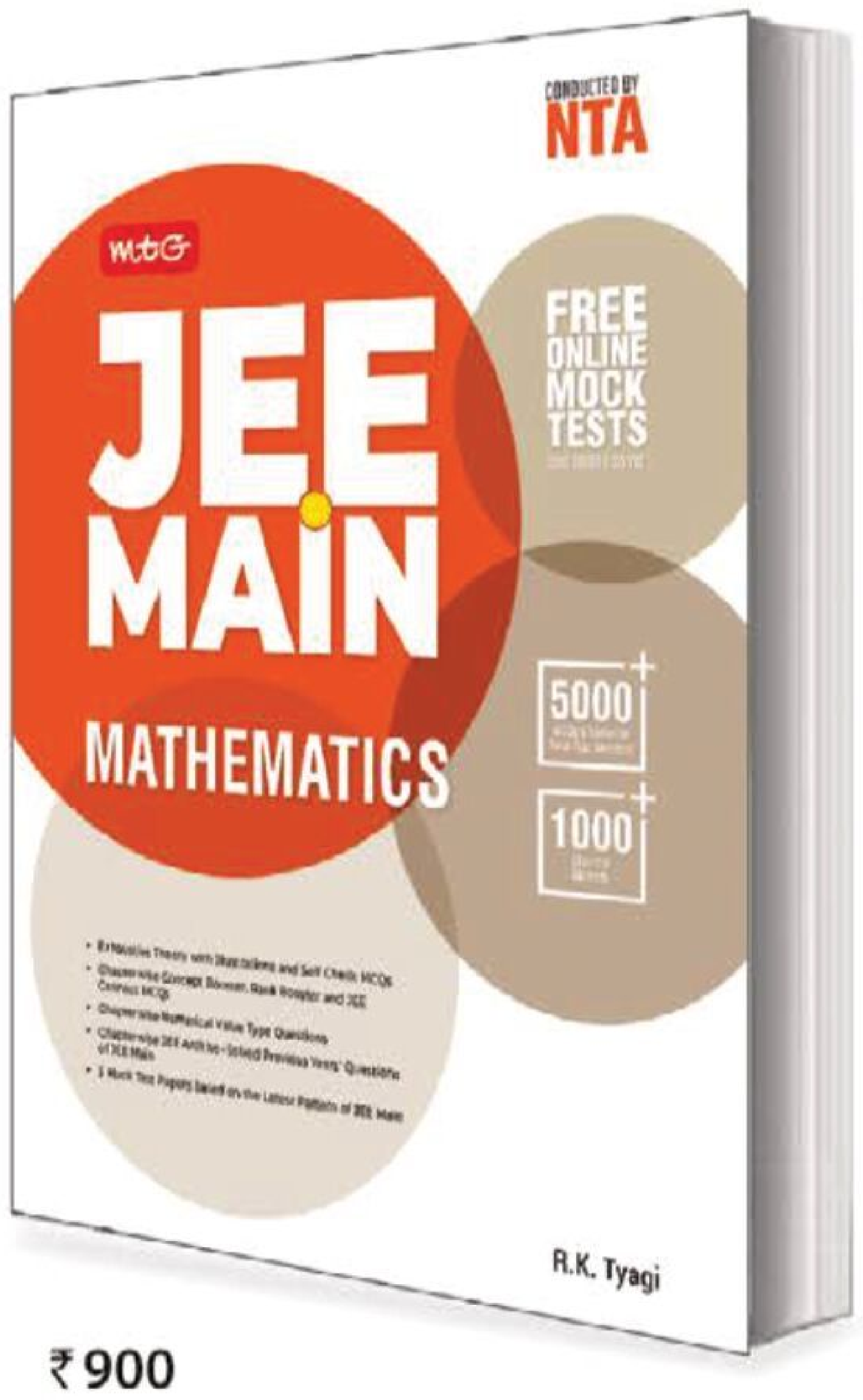
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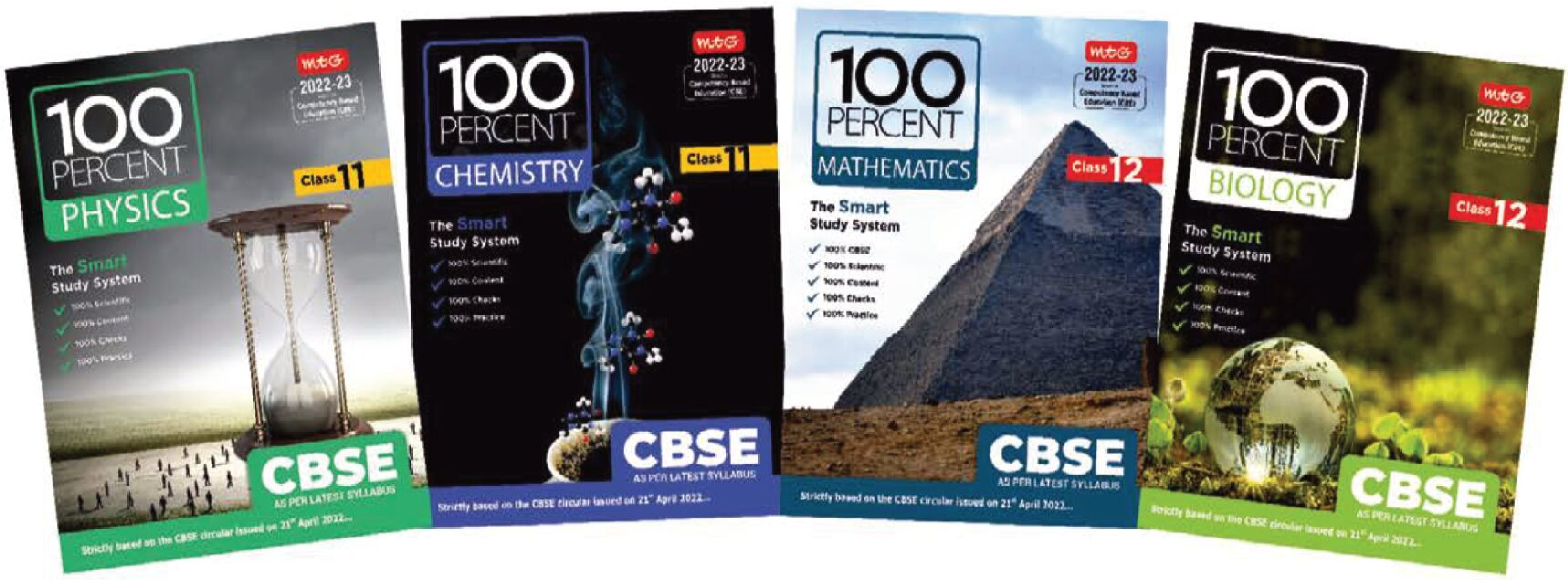
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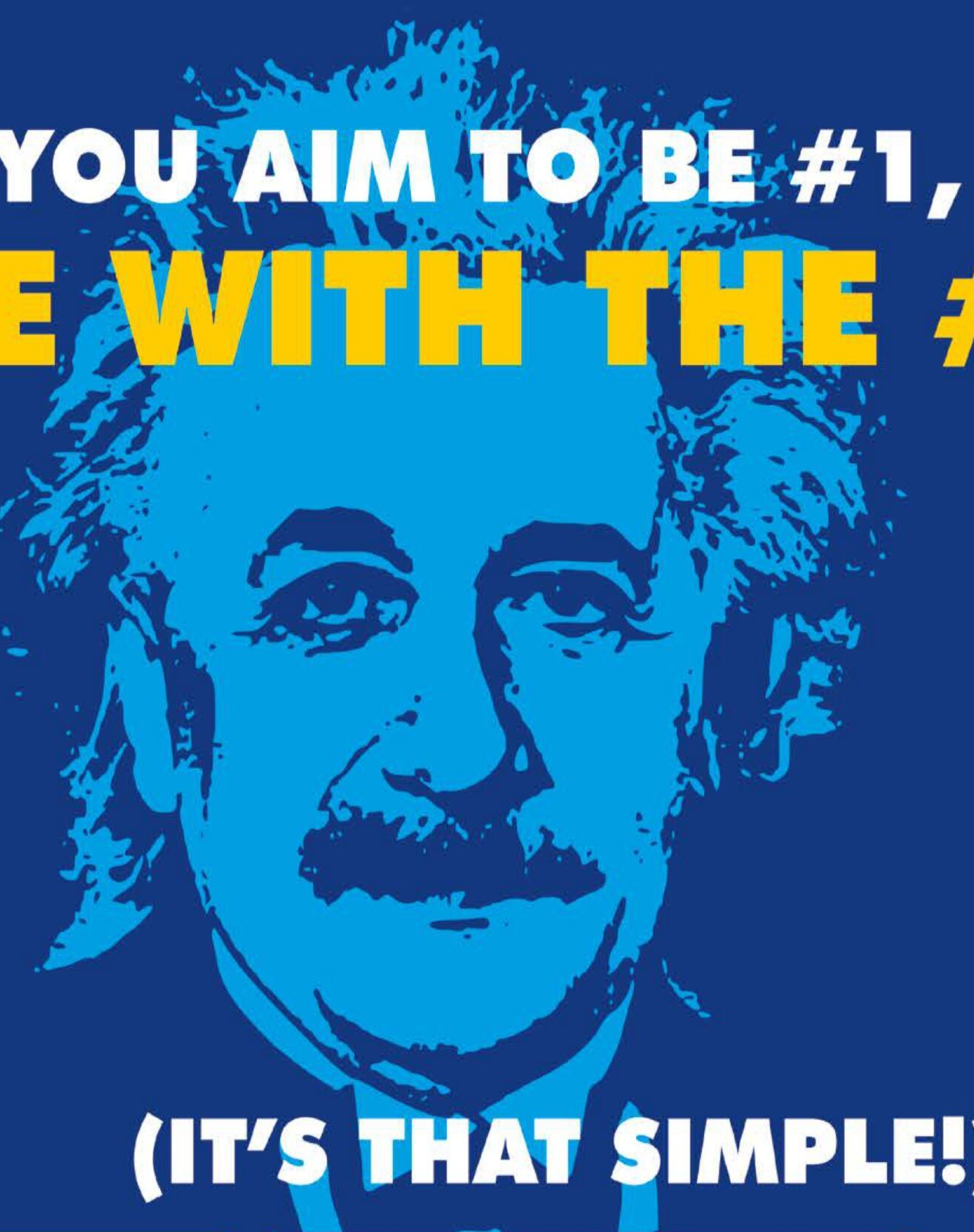

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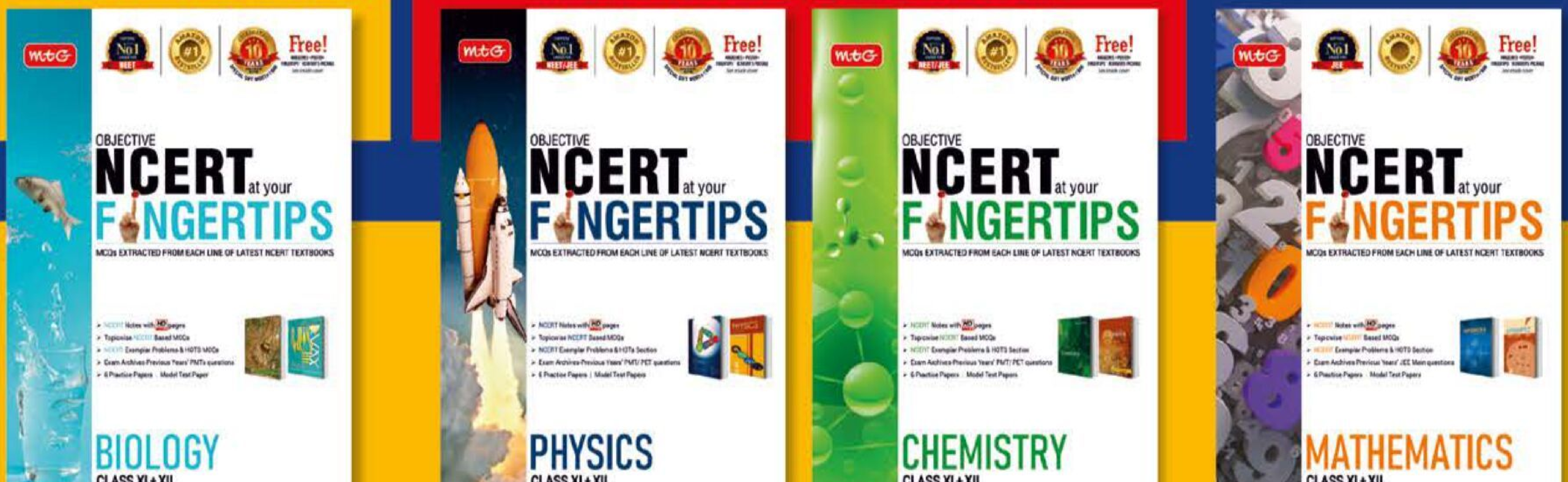
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